Kennesaw State University
College of Science and Mathematics
Department of Mathematics
Fall Semester 2023
MATH 4391 (Section 51): Complex Analysis

(Un)popular Answers to Popular Questions

1. *What is happening when? When are things due?* See the contact information in Section 1 and the calendar in Section 7.

2. *What is my grade?* See the formula at the start of Section 4. Your grades are on D2L.

3. *What and where are the course materials?* See Section 2. Your grades and solutions to problem sets are password-protected on D2L. Everything else (lecture notes, daily reading log, exams, presentation slides, problem sets, portfolio project) is on the course website.

4. *How should I write an email to you?* See Section 5.3.

5. *How should I be using my time?* See Section 5.5 for a workload breakdown.

6. *I can't come to class today. What should I do?* See Section 4.1 for the attendance policy and how to get attendance credit.

7. *All the notes are online. What's the point of coming to class?* Focus and interaction. Being in class lets you *focus* specifically on course material and *interact* with me and your classmates. Low attendance typically correlates to low grades.

8. *Can I turn in a problem set late?* Possibly. See Section 4.2 for the policy on pushing a problem set back.

9. *Why are you being so picky with the formatting of the problem sets?* For your benefit and for mine: to make your work easier for you to reread later in the future and for me to parse for credit. See also Section 4.2.

10. *Can I make up an exam?* Quite possibly. See Section 4.4 for the procedures and policies regarding exam make-ups.

11. *I don't understand what we're doing, and everything feels awful. How can things get better?* First, remember that no feeling lasts forever; second, don't make major decisions when you're feeling crummy. Start by looking at Section 5. Are you coming to class? Talking with me? Putting in a solid 9 hours of work per week? Try to articulate clearly to yourself and then to me how you're living your Math 4391 life, what your Math 4391 goals are, and what precisely is not right with your Math 4391 experience.

1. Contact Information

Instructor: Dr. Timothy Faver
Email: `tfaver1_AT_kennesaw.edu` (this is the best way to contact me)
Website: `https://tefaver.com/teaching/math-4391`
Lecture time: MWF 10:10 am–11:00 am
Lecture time: Mathematics Room 116
Office: Mathematics Room 248
Office hours: MW 3:30 pm–4:30 pm

2. Learning Outcomes

Upon successfully completing this course, you will be able to do the following:

1. Use properties of elementary functions (trigonometric, exponential, etc.) of a complex variable;

2. Determine if a given function of a complex variable is continuous, analytic, or integrable;

3. Differentiate and integrate functions of a complex variable; use Cauchy's integral theorem, Cauchy's integral formula, and the residue theorem to evaluate elementary contour integrals; prove related elementary theorems.

4. Perform elementary calculations with Taylor and Laurent series;

5. Apply techniques of complex analysis to solve certain applied problems (e.g., compute certain Riemann integrals, determine certain conformal maps or harmonic functions with given boundary values).

Informally, a complex number is an expression of the form $z = x + iy$, where x and y are real numbers and $i^2 = -1$. Making precise sense of the word "expression" and in particular the juxtaposition iy will be one of our first tasks. I see the course dividing into three phases; in each phase, we will see the recurring leitmotifs of *geometry*—the identification of complex numbers $x+iy$ with ordered pairs of real numbers (x, y)—and *algebra*—our ability to multiply complex numbers and get a new complex number.

The initial phase is precalculus: the arithmetic, algebra, and geometry of complex numbers and the definition and properties of fundamental functions defined on complex numbers (polynomials, exponentials, logarithms, and trigonometric functions). The intermediate phase is differential calculus: limits, continuity, and derivatives for functions of complex variables—the first two will proceed very much as they do in multivariable calculus, and the third definitely won't. The major phase, in which we'll spend most of the course, is integral calculus. The integral is *the* tool for extracting useful information about functions, and we will prove and apply many powerful results in the language of integrals.

Along the way, we'll develop a deeper understanding of and appreciation for topics in single and multivariable calculus. We'll even revisit some kindergarten geometry, as it turns out that circles and triangles are incredibly important shapes. While this is not a course in formal proof like your real analysis courses, we will nonetheless prove many results in

rigorous detail (albeit with fewer ϵ–δ chasing); after all, a proof is just an argument that you're correct.

3. Course Materials

1. We will follow the book ***Complex Analysis with Applications*** by Nakhlé Asmar and Loukas Grafakos. Sometimes we will follow the book very closely, and other times we will diverge nontrivially. The book is an incredibly rich source of examples and problems; I caution you that I will not do nearly enough examples, for your taste, or mine, in class, so ityou must develop a close relationship with this book. A solutions manual for selected odd problems is on D2L.

2. I will write **daily lecture notes** for each class and post them on the course website. Lecture notes will contain *everything* that we discuss in class as well as recommended problems for you to work. I encourage you to read and attempt these problems as you read the notes; I have placed them in specific locations to correspond to the covered material.

3. I will maintain a **daily log of readings** from Asmar–Grafakos that correspond to what we covered in class. You should consult this log for additional examples and different perspectives.

4. There are many other good complex analysis books out there, so if you want some additional perspectives on a subject, let me know, and I can give you extra resources. I am particularly fond of *Complex Analysis* (Second Edition) by Freitag and Busam.

5. The materials above are *required* or *recommended*. Here are some *prohibited* materials. Please remove AirPods and other listening devices during class. Please keep phones off your desks, and please do not take pictures of the board without my explicit permission.

4. Grading

Your final numerical grade will be based on your daily attendance, written problem sets, in-class quizzes, two in-class exams, a final exam, and a glossary project. The following weights will determine your final numerical grade.

Attendance	5%
Problem Sets	35%
Portfolio	15%
Lowest exam score	10%
Middle exam score	15%
Highest exam score	20%

Your final letter grade will be determined by the interval to which your final numerical grade belongs.

Numerical grade	**[90,100]**	**[80,90)**	**[70,80)**	**[60,70)**	**[0,60)**
Letter grade	A	B	C	D	F

4.1. Attendance (5%). *Regular and engaged attendance is essential for your learning; failures in my classes are strongly correlated with low attendance or attendance with weak engagement.* I will take attendance on each of the days that we meet and do not have an exam. Each day that you are present will add a point to your final attendance score. I may determine attendance by checking the class roster or by collecting some in-class work (which I will not grade for correctness, just engagement). At the end of the term, your attendance grade will be determined by

$$\min\left\{100 \times \frac{\text{Total number of attendance points}}{\text{Total number of meetings} - 3}, 100\right\}.$$

This allows you to miss several classes without penalty.

An absence may or may not be "excused." Important family commitments, religious obligations, feeling ill, COVID exposure/infection, emergency car trouble, legal issues, an essential work commitment, and university-sponsored athletics, trips, or programs are all sufficient reasons for an excused absence. Making up work in another class, oversleeping, traffic, and vacations (...early Thanksgiving break...) are not.

If you feel that your absence merits an excuse, please read the material covered in the textbook that day and then email me (1) the reason for your absence and (2) a question about something that you don't understand from that day's discussion or, if you feel that you understand everything, a comment about something that seems interesting, or frustrating, or weird, or anything that evokes an emotional reaction in you. To receive the attendance credit, you need to send the email before you next return to class. An excused absence will not receive attendance credit without the component (2).

The university has eliminated the former grade of WF that was sometimes assigned to students who stopped attending class after the withdrawal deadline (but never formally withdrew). If you stop coming to class and do not submit work at some date past the withdrawal deadline, I will score missing work as 0 and compute your final grade from that; it is therefore highly likely that you will fail the class.

4.2. Problem sets (35%). *Regular work on weekly problem sets will strengthen your understanding of course concepts and techniques. Success in this class will be impossible without diligent, thorough completion of the problem sets.* There will be 12 problem sets assigned on Fridays and due throughout the term at 11:59 pm on the following Fridays; see the calendar (Section 7) for the exact dates.

I will drop your lowest 3 problem set scores, so that your the problem set component of your final grade will be the average of your 9 best problem set scores. I will grade some problems for correctness and the rest for completeness. Problems graded for correctness will be scored from 0 to 5 points, with 0 points for no work, 5 points for essentially complete and correct work, and, in general, a 1-point deduction for each serious error. Problems graded for completeness will be scored from 0 to 2 points, with 0 points for no work, 2 points for complete work, and 1 point for work that, in my professional opinion, falls short of completeness.

You will submit your solutions to the appropriate slot on D2L, and it is your responsibility to ensure that your D2L submissions are correctly formatted and organized according to the submission instructions. The goal of these stringent formatting requirements (to be

repeated/expanded in individual assignment instructions) is twofold. First, the goal is to provide for you a detailed set of examples to which you can return as you study. I want your work to be as easy for you to revisit and parse as possible, and part of that is making sure your work is clearly organized and formatted. Second, the goal is to provide for me an assignment that is easy to review so that you get appropriate credit and so that I am sure you are doing the work that is meant to strengthen you.

You may push back three assignments from their Friday due date until the following Monday. For example, if you wish to push Problem Set 1 back, you would have until Monday, August 28, at 11:59 pm to submit it. You do not need to provide any reason for a pushback. However, you must request a pushback no later than 5 pm on the Thursday before the assignment is due. For example, if you wish to push Problem Set 1 back, you would have until Thursday, August 24, at 5 pm to tell me that you want to do so. The point of this condition is to ensure that you have started your assignment in a timely fashion and have been thinking about it throughout the week; leaving problem sets for Friday afternoons is dangerous and deleterious both for your learning and your peace of mind. Once you have used your three pushbacks, I will typically not allow any more, outside of exceptional circumstances. Also, once you have pushed a problem set back, I will not accept it late beyond the Monday deadline. There is no reward or extra credit for *not* using your pushbacks.

You are most welcome to consult with me throughout the week about the preparation of your problem sets; see Section 5.2 for advice on office hours and Section 5.3 for email policy.

4.3. Portfolio (15%). *You will submit a "portfolio" of 10 curated problems from the lecture notes and textbook that will take you beyond focused weekly problem sets and help you make connections across and beyond the course and develop your writing and presentation skills.* Full details of the portfolio project are on the course website; note that you will submit drafts of some problems along with exam reflections to keep you on track. See the course calendar (Section 7) for the portfolio due date and reminders about exam reflections.

4.4. Exams (45%). *Exam preparation will help you connect discrete course topics; completing an exam under time constraints is one way of evaluating your fluency with course material and your intellectual independence.* There will be two in-class exams (Exams 1 and 2) during the term and a cumulative final exam. The final exam will only be given at the university-specified time, which cannot be changed. See the calendar (Section 7) for the exam dates. I will provide a detailed study guide at least a week in advance of each exam. You will earn 5 points on each exam from a reflection activity that you will do based on the study guide; you can earn the other 95 points on the actual exam.

In order to be excused from an exam, you must (1) notify me before the start of the exam (or as soon after as the circumstances allow) and (2) provide official documentation (a doctor's note, a quarantine notice or proof of COVID test, a note from your employer, etc.) excusing your absence in a timely manner. If your absence from an exam is excused, then you will take a make-up exam during the final exam conflict period; if your absence is not excused, you will score a 0 on that exam. Dates and times for the make-up exam are also on the calendar (Section 7). Valid (and invalid) reasons for missing an exam are, in general, the same as for missing class. In general, I will not permit a make-up if you notify me of your absence *after* the class has taken exam, barring exceptional circumstances that make

prior communication absolutely impossible.

In the (hopefully) unlikely event that you first have an excused absence from Exam 1 and later have an excused absence from Exam 2, then you will take a make-up for Exam 2 within one week of returning to class. Failure to do so will result in a score of 0 on Exam 2. This is to prevent you from having too much work at the end of the term.

To motivate and reward long-term improvement over the semester, your final exam grade can replace the lower of your grades for Exams 1 and 2 if the final exam grade is higher. This option is not available if you miss and do not make up Exam 1 or 2; in the unlikely event that your scores on Exams 1 and 2 are the same, the final, if higher, will replace only one of those scores. (To see how this would work, suppose that you score 90 on Exam 1, 70 on Exam 2, and 80 on the final exam. The score of 80 from the final exam would then replace the 70 from Exam 2. Your lowest exam score would then be 80, your middle exam score would also be 80, and your highest exam score would be 90.)

4.5. Midterm grades. I will submit midterm grades by October 3 to help you assess your progress in the course. Here is the official policy: *A midterm grade will be assigned by the midterm grade due date identified on the Fall 2023 academic calendar. This midterm grade is for assessing mid-semester performance prior to the last day to withdraw without academic penalty. You may view your midterm grade in Owl Express. Note that only your final grade will be officially recorded on your academic transcript.*

Your midterm numerical grade will be determined by the following breakdown:

$$10\% \text{ attendance} + 45\% \text{ problem sets} + 45\% \text{ Exam 1}.$$

I will not drop any absences, problem sets, or quizzes in this calculation. Note that the weights here are, overall, quite different from what will determine your final grade; a good midterm grade is not a promise of a good final grade, but a bad midterm grade should serve as a warning that something is amiss in your current approach to the course. Midterm letter grades will follow the numerical correspondence for your final grade.

4.6. Incomplete grades. The catalogue specifies that an "incomplete grade may be awarded only when the student was doing satisfactory work prior to the last two weeks of the semester but for nonacademic reasons beyond the student's control, was unable to meet the full requirements of the course." I define "satisfactory work" to mean that you can obtain a score on your remaining work that will allow you to pass the class.

The catalog subsequently states that a "grade of 'I' must be removed by completing the course requirements within one calendar year from the end of the semester in which the 'I' was originally assigned. In addition, should the student enroll in classes at KSU during the calendar year, the grade of 'I' must be removed by the end of the first semester of enrollment during that calendar year."

If you are awarded an incomplete for this course, it will be to your advantage to complete all remaining work as soon as possible. Based on your circumstances, we will agree on a mutually convenient and appropriate timeline for completing your work. Failure to follow this timeline without a justifiable excuse may result in failing grades for the missed work and possibly the course. Ideally, you would complete all work by the end of January 2024.

5. Strategies for Success and/or Failure

5.1. Attendance and participation. Failures in my classes are strongly correlated with low attendance *or* attendance with minimal engagement; students who both attend class regularly and engage with me in and/or outside class tend to succeed.

5.2. Office hours. During office hours, we can discuss specific examples, problems, or techniques from class, the lecture notes, or the textbook. More broadly, we can talk about your study habits, time management, and mathematical reading skills. Before coming to office hours, think carefully about what you want to discuss so that we use our time well.

If my office hours conflict with your other classes, we can make an appointment for a different time. If the currently scheduled office hours conflict with the availability of students who most frequently want to see me, I may change the weekly hours to accommodate the preferences of the majority.

An online option via Teams will always be available for office hours. The Teams link will be on D2L. I will attempt to hold office hours in person (I encourage masking).

5.3. Email communication. You are welcome to email me questions about any aspect of the course at any time. Feel free to send a photo or scan of work and indicate where you're stuck. A short hint from me can make a big difference for you.

I will (typically) respond to email within 24 hours, but I cannot guarantee a response to questions about problem sets or exams late on Thursday evenings. Those final hours before an exam or quiz need to be about perfecting what is already good in you, not about learning it for the first time.

Please observe the following courtesies in writing emails.

1. Please include the words "Math 4391" in the subject of your email. I filter emails and may not respond to your message promptly, or may miss it entirely, without these key words.

2. Please use your "students" email, not D2L email. I will not respond to messages sent from D2L.

3. Please begin your email with a salutation (e.g., "Hi, Dr. Faver") before the body of text; this is simply good manners (and will put me in a good mood, which will be helpful for you). Please also address me by my name ("Dr. Faver," not just "Professor").

5.4. Reading assignments and recommended problems. It is quite likely, and wholly natural, that you will not fully understand the material that we cover in class. Readings from the textbook(s) will offer different perspectives that may be more conducive to your personal engagement with Math 4391. Part of your mathematical education is developing your reading comprehension; be prepared to invest time, and struggle, with the course's *written* materials before seeking outside help or alternate media.

Doing the problems that appear in the lecture notes and the problem sets *alone* probably will not provide you with enough exposure to course material outside of class. In addition, you should attempt a variety of problems from the "Recommended Problems" lists that I provide each week; challenge yourself to do a certain number of these each week along with the problem sets. Recommended problems will also serve as additional, essential practice

and review for exams. You should skim the textbook's problems that I don't assign, as well, and attempt some of them as you study.

5.5. Workload breakdown. In general, it is reasonable (and usually necessary) for you to spend about three hours per credit hour outside of class on work for a course. Since our course is 3 credits, you should spend about 9 hours per week on your coursework. Here is a recommended arrangement of your worktimes for this class for weeks when a problem set is due, which will be most weeks.

2–3 hours: *active reading of lecture notes and selections from textbook (see the daily log).* I define "*active* reading" to mean reading each and every word (possibly *aloud*—this always helps me), clarifying any unknown word or phrase, and redoing each calculation or argument with pen and paper. Since, presumably, you have been actively engaged in class, your active reading of the lecture notes should not take too much time. Instead, I expect that you will be solving the (!)-problems in the lecture notes (see the introduction to the notes for the different problem types) and working through the material in the textbook selections, which will generally be different from our precise work in class.

3–4 hours: *solving the required problems from the weekly problem set.* I expect you to do whatever you need to in order to understand every step of the solution to every problem. This timeframe by no means expects that you are spending 3 to 4 hours in isolated contemplation of the required problems. Indeed, this will be a highly *nonlinear* activity: you should complete the textbook reading for a particular section before attempting the problems required from that section, but then you should return to the selections from the textbook and the lecture notes (which you have already actively read) corresponding to those problems for ideas and advice. If you are stuck on a required problem, you may also want to attempt a similar-looking recommended problem; often that can give you the key insight for the required work. Of course, you can and should communicate with other people as part of the process of solving the problems, including your classmates and me.

1–2 hours: *writing up your solutions to the weekly problem set for submission.* This really means *rewriting* your initial work. Your initial work on required problems should not, in general, be what you submit to D2L, but by the time that you really understand your solution to a problem, your understanding should be sharp enough that organizing your submitted solution should not be too much of an ordeal.

This scheme will vary from week to week and person to person. It suggests at least 6 hours of work per week, but there will probably be weeks during which you are able to do all of the above without taking 9 hours. Treasure those weeks! You might work on one of the projects during the remaining time, or attempt some of the ($\star$)-problems that were not assigned on problem sets, or revisit prior work, or put in some extra work on the portfolio project. It will be to your benefit to spread these activities out throughout the week.

5.6. How to fail. The following strategies have helped prior students fail my classes.

1. Don't come to class regularly; in particular, show up only for exams.

2. Be physically present but intellectually disengaged in class. Avoid talking with me.

3. Don't do the problem sets or any recommended practice problems.

4. Save all your Math 4391 work for only one day each week, preferably the day before assignments are due.

5. Ask for, but then completely ignore, my advice on improving your course performance.

6. Think about the course as an obstacle, not an opportunity. Definitely don't look for beauty, surprises, or things that might fill you with wonder and curiosity.

6. University Policies and Formal Matters

6.1. Federal, BOR and KSU Student Policies. You should be familiar with the policies detailed via the link below.

`https://cia.kennesaw.edu/instructional-resources/syllabus-policy.php`

6.2. KSU student resources. You should be familiar with the resources and rights available to you as detailed in the link below.

`https://cia.kennesaw.edu/instructional-resources/syllabus-resources.php`

6.3. Academic integrity statement. Every KSU student is responsible for upholding the provisions of the Student Code of Conduct, as published in the Undergraduate and Graduate Catalogs. Section 5c of the Student Code of Conduct addresses the university's policy on academic honesty, including provisions regarding plagiarism and cheating, unauthorized access to university materials, misrepresentation/falsification of university records or academic work, malicious removal, retention, or destruction of library materials, malicious/intentional misuse of computer facilities and/or services, and misuse of student identification cards. Incidents of alleged academic misconduct will be handled through the established procedures of the Department of Student Conduct and Academic Integrity (SCAI), which includes either an "informal" resolution by a faculty member, resulting in a grade adjustment, or a formal hearing procedure, which may subject a student to the Code of Conduct's minimum one semester suspension requirement.

6.4. Course catalogue description.
3 Class Hours 0 Laboratory Hours 3 Credit Hours
Prerequisite: MATH 2203.
This course is an introduction to the basic concepts of complex analysis, its beautiful theory and powerful applications. Topics covered will include: the algebra and geometry of the complex plane, properties of elementary functions of a complex variable, analytic and harmonic functions, conformal mappings, continuity, differentiation, integration (Cauchy integral theory), singularities, Taylor and Laurent series, residues and, time permitting, their applications.

6.5. COVID-19 statements. You should be familiar with the most recent university policies and resources regarding COVID-19 as detailed in the link below.

`https://www.kennesaw.edu/coronavirus`

7. Calendar

You are responsible for knowing all of the due dates for assignments and scheduled dates for quizzes and exams listed below. All submissions to D2L are due at 11:59 pm on the stated day. If class is canceled on the day of a quiz or exam, we will take that quiz or exam during our next meeting, unless I specify otherwise.

F August 25	Problem Set 1 due on D2L
F September 1	Problem Set 2 due on D2L
M September 4	No class—Labor Day
F September 8	Problem Set 3 due on D2L
F September 15	Problem Set 4 due on D2L
M September 18	Exam 1 reflection due on D2L
F September 22	Exam 1 in class
F September 29	Problem Set 5 due on D2L
T October 3	Midterm grades submitted
F October 6	Problem Set 6 due on D2L
T October 10	Last day to withdraw without academic penalty
F October 13	Problem Set 7 due on D2L
F October 20	Problem Set 8 due on D2L
M October 23	Exam 2 reflection due on D2L
F October 27	Exam 2 in class
F November 3	Problem Set 9 due on D2L
F November 10	Problem Set 10 due on D2L
F November 17	Problem Set 11 due on D2L
M–F November 20–24	No class—Thanksgiving Break
T November 28	Final exam reflection due on D2L
F December 1	Problem Set 12 due on D2L
M December 4	Portfolio due on D2L
T December 5	Exam make-ups, 3:30 pm–4:30 pm
M December 11	Final exam, 10:30 am–12:30 pm

Week	**Content**
1	Basic arithmetic and geometry in $\mathbb{C}$
2	Functions of a complex variable, sequences
3	Series, exponential, trigonometry
4	Polar coordinates, arguments, logarithms
5	Complex powers, algebra, exam review
6	Limits, basic topology, continuity
7	Removable discontinuities, differentiation
8	The Cauchy–Riemann equations, connectedness
9	Paths and definite integrals, exam review
10	Line integrals, independence of path
11	The Cauchy integral theorem and formula
12	Power series and complex analyticity
13	Zeros and singularities
14	Laurent series, the residue theorem
15	Residue calculus, final exam review