**Day 1: Monday, August 14.** We covered the elementary calculations on pp. 2–3. Definition 1.1.1 gives a much better definition of "complex number" than we did; we will revisit that in a few days. We did not discuss Section 1.1 in the lecture notes, but you should read that for background on set-theoretic terminology.

**Day 2: Wednesday, August 16.** We continued with arithmetic as on pp. 4–7, up to and including Proposition 1.1.5. These pages have many more examples of arithmetic than we did in class (and maybe even some solutions to problems posed in the lecture notes). Then we looked at the complex plane geometrically as on p. 13 and the modulus as on pp. 15–16 (omit Example 1.2.4). Also read Proposition 1.2.5, Remark 1.2.6, Example 1.2.7, and Proposition 1.2.8 (omit the geometric proof). Examples 1.2.9, 1.2.10, and 1.2.11 are helpful models of inequality manipulations, but we will do very little of that in this class (and I will require you to do none of it yourselves on problems, unless you want to).

**Day 3: Friday, August 18.** The book defines complex numbers as ordered pairs in Definition 1.1.1 but otherwise doesn't use the ordered pair language as we have. After next week, we will never use the ordered pair language again, either, but contemplating the abstract construction of $\mathbb{C}$ is an important step at least once in your mathematics education. You should read the field axioms in the lecture notes and skim Example 1.3.2 to see how the axioms are used. However, I will never make you work with those axioms. Likewise, you should spend a bit of time reading about completeness in the lecture notes, but, again, I won't require you to do problems with it. If you can articulate that the real numbers are a set in which arithmetic and comparison work as they should and in which there are no gaps, then you have enough grasp of their abstract structure for my purposes.

**Day 4: Monday, August 21.** We talked about functions, which we will continue to do for the rest of the course. Pages 41–47 give examples of a variety of functions and include visualizations of domains and ranges. Since we have not yet studied polar coordinates, you may only want to consider Examples 1.4.1, 1.4.4, and 1.4.5 right now.

**Day 5: Wednesday, August 23.** We talked about extensions and restrictions of functions, which the book does not discuss explicitly. Be sure that you are comfortable with the definitions and notation from the lecture notes. Then we started sequences: see pp. 52–53 for definitions and notation, Example 1.5.3 for a basic convergence argument, and Theorem 1.5.7 for the essential algebra of sequences.

**Day 6: Friday, August 25.** We finished talking about sequences. See Fig. 1.31 for an illustration of sequential convergence in the plane, and also Fig. 1.32. We proved the "conjugate" part of Theorem 1.5.7 and the forward direction of Theorem 1.5.8. The book's proof of Theorem 1.5.8 is different and well worth reading, as it uses the clever identity $\text{Re}(z) = (z + \overline{z})/2$. Example 1.5.9 is a helpful result with a technical proof; understanding this proof is a good measure of understanding sequence manipulations. We will not discuss Cauchy sequences in this class, so you can omit Definition 1.5.10 and Theorem 1.5.11 (if you promise to take real analysis). Finally, we discussed series on p. 56; our definition is rather more precise than the book's "expression of the form."

**Day 7: Monday, August 28.** We continued talking about series. Our major result was the geometric series; see Theorem 1.5.13 and Example 1.5.14. Example 1.5.9 discusses the convergence and divergence of the "power" sequence $(z^k)$ in a manner somewhat different from our treatment of this sequence during the geometric series proof. Note the philosophy in the paragraph preceding Theorem 1.5.15 and then be prepared to use that theorem frequently. Everything on pages 59 and 60 is worth reading, although you are not required to know the proofs of Theorem 1.5.17, Proposition 1.5.18, Theorem 1.5.20, or Theorem 1.5.21. (All demonstrate valuable techniques in *analysis* but nothing unique to *complex analysis*.)

**Day 8: Wednesday, August 30.** We stated the ratio test (Theorem 1.5.23) and applied it to the exponential (Example 1.5.24). The root test (Theorem 1.5.25) is also good to know. Then we started talking about the exponential, which the book calls $e^z$ almost exclusively. (We have yet to earn the right to take complex powers, so I will say $\exp(z)$ in class.) Theorem 1.6.2 proves the "functional equation" $\exp(z + w) = \exp(z)\exp(w)$ using the material about Cauchy products on pp. 62–63; this is nice to know but optional. Pages 66–68 develop some computational properties of the exponential that I will ask you to do as part of an upcoming problem set; the book presumes more about the trig functions than I want us to. Page 76 defines the complex sine and cosine in terms of the exponential, as we did, and presumes that $\pi$ works as it should. I am approaching $\pi$ (and everything else) in a slightly more fundamental way to illustrate what are the most basic assumptions that we need about these things. We will eventually fill in much of the material in Sections 1.6 and 1.7 in different orders from the book; you should consult these sections as you need to.

**Day 9: Friday, September 1.** We discussed a variety of topics related to the complex exponential, sine, and cosine. You may want to read Proposition 1.6.3, Corollary 1.6.4, Example 1.6.5, Proposition 1.6.7 (and the paragraph preceding it on p. 68) and then Example 1.7.2, Propositions 1.7.3 and 1.7.4, and Examples 1.7.5 and 1.7.7. Solutions to some of the problems in the lecture notes can be found in these areas, and you are welcome to consult them as you work on required or recommended problems from the notes.

**Day 10: Wednesday, September 6.** We went back and finally discussed polar coordinates. The book does this in Section 1.3 by assuming that polar coordinates exist as we did in Calculus II. (In contrast, in class we *proved* the existence of polar coordinates by assuming some other fundamental things.) All of the material on pp. 25–30 is worth reading. Note that (1.3.13) gives a formula for the principal argument. . . if you believe in the inverse tangent first. We will discuss other parts of Section 1.3 later. You should also read p. 68, which discusses polar coordinates in the context of the exponential and then the section "Exponential and Polar Representations" on pp. 69–72 (you don't have to read Example 1.6.10 yet).

**Day 11: Friday, September 8.** We developed the complex logarithm, as on p. 86 and Definition 1.8.2 and Definition 1.8.4. Note that the book is not typically using set-builder notation with the symbols log and arg. Examples 1.8.1, 1.8.3, and 1.8.5 give lots of good computational practice. Think carefully about the observations at the top of p. 88.

**Day 12: Monday, September 11.** Pages 90–91 discuss complex powers (stop on p. 90 with the paragraph ending "As a convention, $e^z \ldots$"). Read Example 1.8.7 and the three cases starting at the bottom of p. 90.

**Day 13: Wednesday, September 13.** We used polar coordinates and the complex exponential to study the algebraic equation $z^n = w$, which the book does way back on pp. 34–36, albeit without the exponential. Read Examples 1.3.11 and 1.3.12 and look at the pictures (Figures 1.20 and 1.21). Then we thought more broadly about functions. See pp. 45–47 on the real and imaginary parts of a function; in the examples, you can skip the geometric discussions (unless you're interested) but do consider the algebraic manipulations of writing $f(x+iy) = u(x,y) + iv(x,y)$ and figuring out $u$ and $v$. We will be discussing limits, continuity, and derivatives starting in Section 2.2 for the foreseeable future.

**Day 14: Friday, September 15.** We developed the correct definition of limit for a function (via sequences) and talked about some useful (but unsurprising) consequences of that definition. The book does this via the $\epsilon$-$\delta$ definition, which we will study later. All of the material on pp. 103–106 is worth reading, and we will eventually cover all of it. Figure 2.8 on p. 103 illustrates limits geometrically, and we will use this idea when we introduce topology next week. You may want to skim Section 2.1 now; this is a sort of catch-all section for topological concepts, but we will meet them more slowly, and only as we need them.

**Day 15: Monday, September 18.** We continued discussing limits and started breaking them. Example 2.2.15 presents limits of rational functions in the context of continuity (which we will discuss shortly), and Example 2.2.17 corresponds to our limits of the principal argument, again using continuity vocabulary. You should be able to follow these examples now without too much trouble, but we will build the machinery of continuity from scratch soon.

**Day 16: Wednesday, September 20.** We finished an example from Monday and reviewed for the exam. There is no new reading for today.

**Day 17: Friday, September 22.** You took an exam; what fun.

**Day 18: Monday, September 25.** We briefly discussed limits on $\mathbb{R}$ (for which you should see the lecture notes) and then introduced some fundamental topological tools for limits, and for more things to come. The book discusses open balls on p. 96 with slightly different notation and closed balls on p. 98. The $\epsilon$-$\delta$ definition of limit appears on p. 103 with a useful illustration in Figure 2.8. Continuity is defined on p. 108.

**Day 19: Wednesday, September 27.** We explored properties of continuous functions. Theorem 2.2.13 contains the essential algebraic properties of continuity. Examples 2.2.14 and 2.2.15 contain unsurprising continuity properties of polynomial and rational functions. Example 2.2.16 gives another nonremovable discontinuity, and Example 2.2.17 treats the argument. Examples 2.2.19 and 2.2.20 and Theorem 2.2.21 are worth reading and closely resemble our work in class.

**Day 20: Friday, September 29.** Section 2.3 discusses the derivative. We will strictly avoid using the word "analytic" as a synonym for "differentiable" for quite some time. All of the material on pp. 114–118 is worth knowing and, up to and including Proposition 2.3.8, hopefully wholly unsurprising. The methods of Example 2.3.9 for proving that a function is not differentiable are important. You should know the statement of the chain rule in Theorem 2.3.11, but you do not need to know its proof, nor the proofs of any of the prior differentiation rules. We will discuss the reverse chain rule, however.

**Day 21: Monday, October 2.** Theorem 2.3.12 contains a slightly different (and maybe more general) version of the reverse chain rule. Look at Figure 2.14 for a reminder of how the order of composition goes here. Example 2.3.13 is worth working through in its entirety. Can you get this result using just the reverse chain rule in the lecture notes? The book subsequently treats complex-valued functions of a real variable separately on pp. 143–144. I claim that this separation of coverage is not really necessary and that our work in the lecture notes is sufficient to allow the domains $\mathcal{D}$ to be a subset of $\mathbb{R}$ or of $\mathbb{C}$. Finally, you can read about open sets (with some other interspersed details) on pp. 96–98. There's a lot here, but I only expect you to know the material in the lecture notes.

**Day 22: Wednesday, October 4.** We proved the powerful Cauchy–Riemann equations, which the book discusses on pp. 130–132. You should know this discussion, and the corresponding proof in the lecture notes, extremely well. The book states the equations as Theorem 2.5.1. The actual proof of that theorem is the converse: if the CR equations hold, and a bit more, then the function is differentiable. We will not prove that together.

**Day 23: Friday, October 6.** We did Example 2.5.3, which is one of the Cauchy–Riemann consequences. I suggest skipping the next two examples, but do check over the algebra in Example 2.5.6. Theorem 2.5.7 and Corollary 2.5.8 resemble our results on locally constant functions, except there the function's domain is strengthened to be a "region" (see Section 2.1). We will revisit these results when we have developed more topological machinery later.

**Day 24: Monday, October 9.** We started talking about paths. The book reviews parametric curves first on pp. 139–143 (and we will do some of these examples very soon) and then discusses paths (a specialization of parametric curves) on pp. 145–148 (again, we will do some of these examples soon). There is a mistake in Definition 3.1.9: it should read "$f$ is continuously differentiable," not "$f'$ is continuously differentiable."

**Day 25: Wednesday, October 11.** Example 3.1.2 presents the incredibly important circles, line segments, and arcs as paths. Definition 3.1.4 defines the reverse of a path, and Example 3.1.5 does the reverse of a line segment. Definition 3.1.11 defines composition of paths. The other examples on pp. 146–148 are worth reading. Equivalent parametrizations appear in Definition 3.2.16 a bit later.

**Day 26: Friday, October 13.** We introduced the tool of connectedness to get the answer we wanted for the ODE $f'(z) = 0$. This appears in the book on pp. 99–100. The book's definition is slightly more restrictive: the path must be polygonal (= composition of line segments), and so we might call the book's definition "polygonal connectedness." Polygonal connectedness implies our definition of connectedness, and the reverse turns out to be true. We essentially proved Theorem 2.5.7 in class.

**Day 27: Monday, October 16.** At last we started integrating. All of the material on pp. 149–153 regarding definite integrals and antiderivatives is worth reading. We will not use the book's "continuous antiderivative" terminology; any antiderivative for us will have to be continuous, because it will be differentiable. See also Definition 3.3.1 and Example 3.3.2 for antiderivatives of functions of a complex (and not necessarily real) variable.

**Day 28: Wednesday, October 18.** We proved the fundamental theorem of calculus! The book discusses this on pp. 153–155, although this treatment assumes the FTC for real-valued functions and extends it from that to the complex-valued case.

**Day 29: Friday, October 20.** A kind visiting instructor introduced the complex line integral. All of the results and examples on pp. 156–161 are worth reading carefully. There are many more examples here than we will do in class. Note that the book writes its integrals over circles as

$$\int_{|z-z_0|=r} f(z)\ dz = \int_{C_r(z_0)} f(z)\ dz.$$

Also, in Example 3.2.13, I would never use $x$ in the integrand that way, so just think of the integral as

$$\int_{|z|=1} \mathrm{Re}(z)\ dz.$$

Finally, a good argument for why the arc length formula is what it is appears on pp. 161–162; see also Example 3.2.18.

**Day 30: Monday, October 23.** We proved the ML-inequality, as on p. 163. See it in action in Example 3.2.20. We then proved (part of) the independence of path theorem. Specifically, we proved that (b) implies (a) in Theorem 3.3.4. This is the hardest part of the proof, and we will work with the other parts next time.

**Day 31: Wednesday, October 25.** We finished proving the independence of path theorem, i.e., the rest of Theorem 3.3.4 in the book. The remaining examples on pp. 171–175 are amazing, and you should do all of them. Note that some are really the fundamental theorem of calculus for line integrals, which the book includes as part of Theorem 3.3.4, but which we did separately, and earlier.

**Day 32: Friday, October 27.** Exam 2.

**Day 33: Monday, October 30.** Section 3.4 proves a version of the Cauchy integral theorem using line integrals from vector calculus. We will not touch this at all. For future reference, it may be worth reading pp. 177–178 to see the definitions of interior and exterior of curves and positively and negatively oriented curves. The book uses this language a lot, but we won't in class.

Section 3.5 gives a different proof that also differs from our work in class. This proof hinges on the special case of Theorem 3.5.2, which requires a topological property from compactness (p. 184). Assuming this special case to be true, you should be able to follow the proof of Theorem 3.5.4, which resembles some of the material in the appendices to the lecture notes.

Section 3.6 gives a deeper generalization of Cauchy's theorem in part (iii) of Theorem 3.6.5 and in Theorem 3.6.7. These results require more topology and analysis than are appropriate for our course. In particular, they introduce the notion of "homotopy," a word I've said a few times in class.

You definitely don't have to read Section 3.4 (although Examples 3.4.7 and 3.4.8 are useful), and you are not obligated to read Section 3.5 or 3.6 or 3.7. The version of Cauchy's integral theorem in Theorem 3.5.9 in the lecture notes will be all that we need. However, the material on pp. 184–185 on star-shaped sets may be helpful.

**Day 34: Wednesday, November 1.** See Examples 3.4.7 and 3.4.8 for useful Cauchy consequences. The material on logs revisited is not in the book, so you will need to rely on the notes.

**Day 35: Friday, November 3.** Today's material is not in the book, so you will need to rely on the notes.

**Day 36: Monday, November 6.** We proved the magnificent Cauchy integral formula, and there was much rejoicing in the land. The book states and proves this as Theorem 3.8.1. The book's version is vastly more general than ours in that it allows more arbitrary paths than circles, but at the cost of relying on the somewhat ambiguous notions of "positive orientation" and "interior." (Well, intuitively these notions are not ambiguous, but try casting them in exact mathematical language.) We will only use our version of the integral formula over circles—to get the "good stuff" that follows, circles are all we need. (Kindergarten geometry FTW!)

**Day 37: Wednesday, November 8.** Example 3.8.2 is very similar to our first example in class today. For Example 3.8.3, try rewriting the integrand using partial fraction and then use the Cauchy integral formula—no need for Cauchy's theorem "for multiply connected domains." The book states the generalized Cauchy integral formula in Theorem 3.8.6 and proves it using a more general differentiation under the integral argument (Lemma 3.8.4 and Theorem 3.8.5); that argument is more powerful than the one in the notes, but it requires more technical hypotheses on the integrand. Read Example 3.8.7 afterward.

**Day 38: Friday, November 10.** Liouville's theorem is Theorem 3.9.2. It relies on a more general version of the estimate that we proved; this is Theorem 3.9.1 (take $n = 2$ in that theorem for our estimate). Theorem 3.9.4 proves the fundamental theorem of algebra but relies on the notion of the limit of a function as $z \to \infty$ in $\mathbb{C}$ (Definition 2.2.10), which we did not develop. Our series expansion is Theorem 4.3.1, which relies on uniform convergence arguments and the Weierstrass M-test from Section 4.1 (in the book); we will not use those methods, and Problems 4.1.4 and 4.1.5 in the lecture notes outline (somewhat) more self-contained approaches.

**Day 39: Monday, November 13.** Examples 4.3.4, 4.3.6, 4.3.7, 4.3.8, and 4.3.9 offer lots of practice with manipulating Taylor series and with obtaining Taylor series for new functions from known ones. Definition 4.2.1 defines power series, and Example 4.2.2 tests the convergence of power series using the ratio and root tests. Theorem 4.2.5 states precisely the result on the radius of convergence, but doing this precisely requires the notion of a lim sup, which is too much real analysis for our class. You do not need to know this theorem as stated in the book, but you should read Definition 4.2.6 and look at Figure 4.5.

**Day 40: Wednesday, November 15.** We continued discussing power series. Corollary 4.2.9 gives the formula for differentiating a power series, and Remark 4.3.3 revisits this formula for Taylor series. Our work with the Taylor series of the principal logarithm resembles Problem 41 in Section 4.3—see p. 261.

**Day 41: Friday, November 17.** The material on pp. 272–273 corresponds very closely to our discussion of the zeros of an analytic function. We have not yet introduced the notion of an "isolated" zero, but we will. See also Example 4.5.3.

**Day 42: Monday, November 27.** No class.

**Day 43: Wednesday, November 29.** We augmented the previous results about zeros by discussing "isolated" zeros. See the top of p. 273 for the definition and the bottom of p. 273 for the proof that a zero of order $m$ is isolated. Then we proved a more involved version of Theorem 4.5.4 and deduced from that the identity principle (Theorem 4.5.5).

**Day 44: Friday, December 1.** We applied the identity theorem to study the additivity of the principal logarithm. A morally similar application is Example 4.5.6. Then we briefly discussed isolated singularities, which are defined on p. 276. Figure 4.16 is an excellent illustration of the three types, and Example 4.5.9 gives three concrete examples.