Reflection Activity

Submit responses to the following questions to the 2 slot on D2L by 11:59 pm on Monday, March 25. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Why is finding an antiderivative for a generic function $f\colon \mathcal{D} \to \mathbb{C}$, with $\mathcal{D} \subseteq \mathbb{C}$ but maybe $\mathcal{D} \not\subseteq \mathbb{R}$, so much more of a big deal than when $\mathcal{D} \subseteq \mathbb{R}$?

2. (Required.) Submit drafts of solutions to 2 portfolio problems from the range Friday, February 9 to Friday, March 22. (As you will see below, this range of dates actually includes more material than can appear on the exam, so it's okay if you submit portfolio problems involving material not directly related to what I'll test you on.)

3. (Optional.) What would you like to discuss as a class during our review on Wednesday, March 27? Be as specific as you can—refer to a problem from the textbook or lecture notes or a particular example, or theorem, or page in the notes that you would like to review.

Exam Content

The exam will cover material from Monday, February 5 (starting with continuity, which I removed from Exam 1) to Monday, March 18, i.e., Days 12–27. The exam will not test material on the Fourier transform or the Cauchy integral formula.

1. Define the following concepts: smooth path, path, initial or terminal point of a path, closed path, image of path, parametrization of a subset of $\mathbb{C}$, line integral, connected set, domain (Definition 3.4.1, not domain of function), path independence, star domain, star-center

2. Determine if a function is continuous at a point and, if not, if the discontinuity is removable.

3. Use the definition of the derivative to prove that a given function is, or is not, differentiable at a given point.

4. Use the reverse chain rule to prove that a function is differentiable.

5. State and prove the Cauchy–Riemann equations in the sense of Problem 2.6.15 in the lecture notes.

6. State and use the Cauchy–Riemann equations to decide where a function is differentiable.

7. Find a parametrization for a given subset of $\mathbb{C}$ in the spirit of Example 3.1.16 and Problems 3.1.17 and 3.1.18 in the lecture notes. You will not need to memorize or use the formulas for composition and reverse of paths.

8. Compute definite integrals of complex-valued functions of a real variable. You would probably need to substitute; you would not need to integrate by parts.

9. Prove the fundamental theorem of calculus (FTC1) for definite integrals—Theorem 3.2.14 in the lecture notes—assuming that Lemma 3.2.15 in the lecture notes is true. If I ask you to do this, I would state Theorem 3.2.14 and Lemma 3.2.15, but I would not tell you any of the properties of definite integrals that you need to use.

10. Compute line integrals via the definition, properties (like those in Problem 3.3.9 in the lecture notes), the FTC for line integrals, or the Cauchy integral theorem.

11. Prove the path independence theorem (Theorem 3.4.4 in the lecture notes) assuming Lemma 3.4.5 in the lecture notes. If I ask you to do this, I would state Theorem 3.4.4 and Lemma 3.4.5, but I would not tell you any of the properties of line integrals that you need to use.

12. Prove (i) $\implies$ (ii) or (ii) $\implies$ (iii) from Theorem 3.4.7 in the lecture notes (this would require you to know how to do Problem 3.3.14 in the lecture notes).

13. Use the Cauchy integral theorem to compute a line integral. You know the value's gonna be 0, your job is to explain why the CIT applies.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?

5. Are you comfortable with defining the various technical concepts that are listed above? Remember that a complete mathematical definition is utterly unambiguous: if you give me a definition of "domain"? I should be able to use it to decide whether any subset of $\mathbb{C}$ I ever meet is in fact a domain. Furthermore, for each definition, can you give a concrete example and/or nonexample? (For "domain," you should be able to write down a particular subset of $\mathbb{C}$ and explain why it is a domain, and you should also be able to write down another subset of $\mathbb{C}$ and explain why it is not a domain.)