Reflection Activity

Submit responses to the following questions to the `Final Exam` slot on D2L by 11:59 pm on Saturday, April 27. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself. Of those 95 points, you can earn 85 from the in-class exam and 10 from the take-home problem announced via email.

1. (Required.) Think about the material that we have covered since Exam 2 (starting roughly on Wednesday, March 20). What topic do you feel least confident about? For at least 30 minutes, reread and rework your notes, my notes, and the book about that topic. Identify terms and notation that you don't understand and isolate steps in proofs or calculations that feel incomplete, unmotivated, or unjustified. Look up those terms/notation and review the work surrounding those difficult steps. How do you feel now?

2. (Required.) The function $f \colon \mathbb{R} \to \mathbb{R} \colon t \mapsto (1+t^2)^{-1}$ is infinitely differentiable but its Taylor series centered at 0 converges only on the interval $(-1, 1)$. Explain why using the geometric series. Next, explain why without reference to the geometric series but instead by thinking about the function $g \colon \mathbb{C} \setminus \{\pm i\} \to \mathbb{C} \colon z \mapsto (1+z^2)^{-1}$. Why can it not have an analytic continuation beyond the ball $\mathcal{B}(0; 1)$? How does this relate to the convergence of the Taylor series for f centered at 0? Theorems 4.1.21 and 4.2.20 in the lecture notes may be helpful.

3. (Optional.) What would you like to discuss as a class during our review on Monday, April 29? Be as specific as you can—refer to a problem from the textbook or lecture notes or a particular example, or theorem, or page in the notes that you would like to review.

4. (Optional.) What advice would you give yourself back in January, or a student taking this class with me in the future, for your (their) success and happiness in the course? Depending on the nature of your response, I may share this with subsequent MATH 4391 classes, so this is a chance for you to help future KSU students!

Exam Content

The final exam is cumulative and will cover material tested on Exams 1 and 2, as well as some new material after Exam 2 as listed below. Some exceptions and caveats appear at the start of the list. In particular, the in-class portion of the final exam will not test material from April 22, 24, or 26. Otherwise, only new material from after Exam 2 appears on this content list; the content lists from Exams 1 and 2 are still fair game.

1. The exam information document for Exam 2 suggested that you prepare proofs of several theorems from the lecture notes. I may ask you for any of those proofs on the final exam, except for the proof of FTC1 (Theorem 3.2.14 in the lecture notes), since I already asked that on Exam 2.

2. The final exam will not ask you about estimating the Fourier transform as in Section 3.5.4 of the lecture notes.

3. You do not need to know how to prove the following results from the lecture notes, or anything about them at all, actually: Lemma 3.5.25, Theorem 3.5.26, Lemma 3.6.16, anything in Section 4.1.2.

4. You should understand the statements of Theorems 3.5.8, 3.5.10, 3.6.8, 3.6.17, 4.1.21, 4.2.20, 4.3.4, 4.3.23, 4.4.6 but you do not need to know their proofs.

5. I may ask you to prove one or more of the following results (I would state it precisely, of course) from the lecture notes: Lemma 3.6.1, Theorem 3.6.23, 4.1.1 (you may assume that you can interchange the sum and integral without any justification), Theorem 4.2.2, Theorem 4.2.12 (you do not need to know how to prove (iv) implies (i)), Corollary 4.2.16, Theorem 4.3.15.

6. You should be able to define the following concepts and give examples and/or nonexamples of them: power series, radius of convergence for a power series, analytic function, analytic continuation, zero of order m of a function, isolated zero of a function, isolated singularity of a function, removable singularity of a function, pole of order m of a function, essential singularity of a function, Laurent decomposition of a function, Laurent series.

7. You should be able to calculate a line integral using the generalized Cauchy integral formula.

8. You should be able to find the Taylor series of a function at a given point and determine the radius of convergence of the series. You should know the Taylor expansions at 0 for e^z, $\sin(z)$, $\cos(z)$, and $1/(1-z)$; you will not need to calculate infinitely many derivatives for the Taylor expansion that I would ask. You do not need to know Taylor expansions for Log.

9. You should be able to find the zeros of a function and determine their order.

10. You should be able to find the isolated singularities of a function and classify them as removable, pole, or essential. You should be able to find an analytic continuation to a removable singularity and determine the order of a pole.

11. You should be able to find both the Laurent decomposition and the Laurent series of a function at an isolated singularity; I will specify the annulus on which you should work. (In this class, the two are not quite the same—see Theorem 4.4.6 in the lecture notes.)

12. You should be able to classify isolated singularities based on the Laurent coefficients.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?

5. Are you comfortable with defining the various technical concepts that are listed above? Remember that a complete mathematical definition is utterly unambiguous: if you give me a definition of "isolated zero," I should be able to use it to decide whether any zero of a function that I ever meet is in fact isolated. Furthermore, for each definition, can you give a concrete example and/or nonexample? (For "isolated zero," you should be able to write down a particular function with a particular zero and explain why it is isolated, and you should also be able to write down another function with a zero and explain why it is not isolated.)