Day 1: Monday, January 8. We discussed how complex numbers arise—naturally, artificially, or cleverly—in problems that seem to involve only real numbers. See pp. 1–2 of the book. I am presuming familiarity with the set theory in Appendix A.1 of the lecture notes. The book discusses some of this on pp. 98–99 starting with the paragraph "Next, we introduce some set notation…"; stop on p. 99 just above "Connected Sets." Appendix B of the notes offers more rigorous perspectives on constructing complex numbers out of real numbers. I will likely assign problems from Appendices A and B for Problem Set 1, and Appendix B also has several portfolio options that you might like.

Day 2: Wednesday, January 10. See p. 2 for one definition of $\mathbb{C}$ as $\mathbb{R}^2$ (which may run into problems since $\mathbb{R} \not\subseteq \mathbb{R}^2$, but we want $\mathbb{R} \subseteq \mathbb{C}$). Pages 3–5 cover elementary arithmetic and pages 13–15 cover elementary geometry. We proved part of Proposition 1.2.5 and will discuss integer powers next time. The many examples on pp. 16–22 are very much worth your time.

Day 3: Friday, January 12. We took another look at division; see pp. 5–7. Proposition 1.1.5 is a useful tool that we will mostly take for granted, and Examples 1.1.6 and 1.1.7 offer helpful additional practice with arithmetic. We then talked about functions, which we will continue to do for the rest of the course. Pages 41–47 give examples of a variety of functions and include visualizations of domains and ranges. Since we have not yet studied polar coordinates, you may only want to consider Examples 1.4.1, 1.4.4, and 1.4.5 right now. See also Appendix A.2 in the lecture notes for some (hopefully familiar) properties of function composition.

Day 4: Wednesday, January 17. We discussed sequences and their convergence and divergence. See Definition 1.5.1, Proposition 1.5.2 for the uniqueness of the limit, and Example 1.5.3 for some concrete sequences and a picture. Example 1.5.9 is a useful result that highlights a variety of different sequence techniques in action.

Day 5: Friday, January 19. See Fig. 1.31 for an illustration of sequential convergence in the plane, and also Fig. 1.32. We proved the "conjugate" part of Theorem 1.5.7 and the forward direction of Theorem 1.5.8. The book's proof of Theorem 1.5.8 is different and well worth reading, as it uses the clever identity $\mathrm{Re}(z) = (z + \overline{z})/2$. We will not discuss Cauchy sequences in this class, so you can omit Definition 1.5.10 and Theorem 1.5.11 (if you promise to take real analysis). Finally, we discussed series on p. 56; our definition is rather more precise than the book's "expression of the form." You should be comfortable with the algebra of series in Theorem 1.5.15 and the test for divergence in Theorem 1.5.17.

Day 6: Monday, January 22. We continued talking about series. Our major result was the geometric series; see Theorem 1.5.13 and Example 1.5.14. Example 1.5.9 discusses the convergence and divergence of the "power" sequence (z^k) in a manner somewhat different from our treatment of this sequence during the geometric series proof. Note the philosophy in the paragraph preceding Theorem 1.5.15 and then be prepared to use that theorem frequently. Everything on pages 59 and 60 is worth reading, although you are not required to know the proofs of Theorem 1.5.17, Proposition 1.5.18, Theorem 1.5.20, or Theorem 1.5.21. (All demonstrate valuable techniques in *analysis* but nothing unique to *complex analysis*.) Last, we stated the ratio test (Theorem 1.5.23) and applied it to the exponential (Example 1.5.24). The root test (Theorem 1.5.25) is also good to know.

Day 7: Wednesday, January 24. The exponential is defined on pp. 64–65, and the book calls it e^z, not $\exp(z)$. We have not earned the right to call it e^z yet. Theorem 1.6.2 proves the functional equation using the Cauchy product formula in Theorem 1.5.28; this is nice to know but not required. Proposition 1.6.3 gives a somewhat euphemistic proof of Euler's identity (with many $\cdots$) assuming knowledge of the real sine and cosine; see also Corollary 1.6.4. This goes in the reverse order of our work in class, in which we defined the sine and cosine in terms of the exponential. Example 1.6.5 offers useful computational practice. Read the last paragraph on p. 68 about π and see Proposition 1.6.7 for the periodicity of the exponential, which presumes periodicity results for the sine and cosine. For trig, see p. 76 for motivation for the definition of the complex sine and cosine and then Definition 1.7.1, Example 1.7.2, and, for more practice, the proofs of Propositions 1.7.3 and 1.7.4 and Example 1.7.7.

Day 8: Friday, January 26. We went back and finally discussed polar coordinates. The book does this in Section 1.3 by assuming that polar coordinates exist as we did in Calculus II. (In contrast, in class we *proved* the existence of polar coordinates by assuming some other fundamental things.) All of the material on pp. 25–30 is worth reading. Note that (1.3.13) gives a formula for the principal argument...if you believe in the inverse tangent first. We will discuss other parts of Section 1.3 later. You should also read p. 68, which discusses polar coordinates in the context of the exponential and then the section "Exponential and Polar Representations" on pp. 69–72 (you don't have to read Example 1.6.10 yet).

Day 9: Monday, January 29. We developed the complex logarithm, as on p. 86 and Definition 1.8.2 and Definition 1.8.4. Note that the book is not typically using set-builder notation with the symbols log and arg. Examples 1.8.1, 1.8.3, and 1.8.5 give lots of good computational practice. Think carefully about the observations at the top of p. 88.

Day 10: Wednesday, January 31. Pages 90–91 discuss complex powers (stop on p. 90 with the paragraph ending "As a convention, e^z ..."). Read Example 1.8.7 and the three cases starting at the bottom of p. 90. The book studies the algebraic equation $z^n = w$ way back on pp. 34–36, albeit without the exponential. Read Examples 1.3.11 and 1.3.12 and look at the pictures (Figures 1.20 and 1.21). Then we thought more broadly about functions. See pp. 45–47 on the real and imaginary parts of a function; in the examples, you can skip the geometric discussions (unless you're interested) but do consider the algebraic manipulations of writing $f(x + iy) = u(x, y) + iv(x, y)$ and figuring out u and v.

Day 11: Friday, February 2. We developed the correct definition of limit for a function (via sequences) and talked about some useful (but unsurprising) consequences of that definition. The book does this via the ϵ-δ definition, which we will study later. All of the material on pp. 103–106 is worth reading; in particular, these pages cover the "algebra of limits" that I put in the notes but did not cover in any detail in class. Figure 2.8 on p. 103 illustrates limits geometrically, and we will use this idea when we connect limits to balls next week. You may want to skim Section 2.1 now; this is a sort of catch-all section for topological concepts, but we will meet them more slowly, and only as we need them. In particular, the concept of accumulation point sneaks in on p. 98. Example 2.2.15 presents limits of rational functions in the context of continuity (which we will discuss shortly), and Example 2.2.17 corresponds to our limits of the principal argument, again using continuity vocabulary. You should be able to follow these examples now without too much trouble, but we will build the machinery of continuity from scratch soon.

Day 12: Monday, February 5. The book discusses open balls on p. 96 with slightly different notation and closed balls on p. 98. The ϵ-δ definition of limit appears on p. 103 with a useful illustration in Figure 2.8. You should read Section 2.2.4 in the lecture notes on limits of functions of a real variable; we did not discuss this in class. Continuity is defined on p. 108. Theorem 2.2.13 contains the essential algebraic properties of continuity. Examples 2.2.14 and 2.2.15 contain unsurprising continuity properties of polynomial and rational functions.

Day 13: Wednesday, February 7. Example 2.2.16 gives another nonremovable discontinuity, and Example 2.2.17 treats the argument. Examples 2.2.19 and 2.2.20 and Theorem 2.2.21 are worth reading and closely resemble our work in class.

Day 14: Friday, February 9. Section 2.3 discusses the derivative. We will strictly avoid using the word "analytic" as a synonym for "differentiable" for quite some time. All of the material on pp. 114–118 is worth knowing and, up to and including Proposition 2.3.8, hopefully wholly unsurprising. The methods of Example 2.3.9 for proving that a function is not differentiable are important. Pages 119–120 briefly discuss local linearity, in particular in Proposition 2.3.10; for a deeper discussion of how the derivative is the "best" local linear approximation, see Section 2.5.2 in the lecture notes. You should know the statement of the chain rule in Theorem 2.3.11, but you do not need to know its proof, nor the proofs of any of the prior differentiation rules. Theorem 2.3.12 contains a slightly different (and maybe more general) version of the reverse chain rule than the one that appears as Theorem 2.5.15 in the lecture notes. Look at Figure 2.14 for a reminder of how the order of composition goes here. Example 2.3.13 is worth working through in its entirety. Can you get this result using just the reverse chain rule in the lecture notes? Last, the book treats complex-valued functions of a real variable separately on pp. 143–145. I claim that this separation of coverage is not really necessary and that our work in the lecture notes is sufficient to allow the domains $\mathcal{D}$ to be a subset of $\mathbb{R}$ or of $\mathbb{C}$; what is really different is how the real and imaginary parts of a function of a *real* variable behave under differentiation, as discussed in Section 2.5.5 of the lecture notes.

Day 15: Monday, February 12. The book discusses the Cauchy–Riemann equations on pp. 130–132 with the assumption that the function's domain is open. We did not make this assumption in our fooling around in class, and we will revisit today's work with that assumption next time. The book discusses open sets on pp. 97–98, and we will discuss them in more detail next time, too.

Day 16: Wednesday, February 14. We discussed some examples and nonexamples of open sets before reviewing for the exam. Part (a) of Example 2.1.3 is our proof that open balls are open. There are some more examples of open sets in the bullet points on p. 98. We will not discuss closed sets or closures or interior/boundary points in our course.

Day 17: Friday, February 16. You took Exam 1.

Day 18: Monday, February 19. We did Example 2.5.3, which is one of the Cauchy–Riemann consequences. Examples 2.5.4, 2.5.5, and 2.5.6 offer more illustrations of using the Cauchy–Riemann equations to check differentiability.

Day 19: Wednesday, February 21. We proved a slightly weaker version of Theorem 2.5.7, which really used Theorem 2.4.7 without the mean value theorem in two dimensions from Theorem 2.4.6. A good review of parametric curves appears on pp. 139–143. The book introduces paths on pp. 145–148. We will avoid discussing piecewise continuous differentiability explicitly, although it will eventually underly the nonsmooth paths that we are constructing. The book defines composition of paths in Definition 3.1.11. Example 3.1.2 presents the incredibly important circles, line segments, and arcs as paths. Definition 3.1.11 defines composition of paths.

Day 20: Friday, February 23. Definition 3.1.4 defines the reverse of a path, and Example 3.1.5 does the reverse of a line segment. The other examples on pp. 146–148 are worth reading. Equivalent parametrizations appear in Definition 3.2.16 a bit later.

Day 21: Monday, February 26. All of the material on pp. 149–153 regarding definite integrals and antiderivatives is worth reading. We will not use the book's "continuous antiderivative" terminology; any antiderivative for us will have to be continuous, because it will be differentiable. See also Definition 3.3.1 and Example 3.3.2 for antiderivatives of functions of a complex (and not necessarily real) variable.

Day 22: Wednesday, February 28. We proved the fundamental theorem of calculus! The book discusses this on pp. 153–155, although this treatment assumes the FTC for real-valued functions and extends it from that to the complex-valued case.

Day 23: Friday, March 1. A good argument for why the arc length formula is what it is appears on pp. 161–162; see also Example 3.2.18. All of the results and examples about line integrals on pp. 156–161 are worth reading carefully. There are many more examples here than we will do in class. Note that the book writes its integrals over circles as

$$\int_{|z-z_0|=r} f(z)\ dz = \int_{C_r(z_0)} f(z)\ dz.$$

Also, in Example 3.2.13, I would never use x in the integrand that way, so just think of the integral as

$$\int_{|z|=1} \mathrm{Re}(z)\ dz.$$

The book states the FTC for line integrals as part (c) of the larger independence of path theorem (Theorem 3.3.4), which we will discuss presently.

Day 24: Monday, March 4. Examples 3.3.5 through 3.3.9 all deal with antiderivatives and use the FTC for line integrals (even though they appear in the section about independence of path). These examples are hugely worth reading and working through in precise detail. Theorem 3.2.19 is the ML-inequality. Read Example 3.2.20. Theorem 3.3.4 contains the major results on independence of path; we only discussed part (a) so far.

Day 25: Wednesday, March 6. We finished proving the independence of path theorem, i.e., the rest of Theorem 3.3.4 in the book. Pages 184–185 discuss star-shaped sets.

Day 26: Friday, March 8. Section 3.4 proves a version of the Cauchy integral theorem using line integrals from vector calculus. We will not touch this at all. For future reference, it may be worth reading pp. 177–178 to see the definitions of interior and exterior of curves and positively and negatively oriented curves. The book uses this language a lot, but we won't in class.

Section 3.5 gives a different proof that also differs from our work in class. This proof hinges on the special case of Theorem 3.5.2, which requires a topological property from compactness (p. 184). Assuming this special case to be true, you should be able to follow the proof of Theorem 3.5.4, which resembles some of the material in the appendices to the lecture notes.

Section 3.6 gives a deeper generalization of Cauchy's theorem in part (iii) of Theorem 3.6.5 and in Theorem 3.6.7. These results require more topology and analysis than are appropriate for our course. In particular, they introduce the notion of "homotopy," a word I've said a few times in class.

You definitely don't have to read Section 3.4 (although Examples 3.4.7 and 3.4.8 are useful), and you are not obligated to read Section 3.5 or 3.6 or 3.7. The version of Cauchy's integral theorem in the lecture notes will be all that we need.

Day 27: Monday, March 18. See Examples 3.4.7 and 3.4.8 for useful Cauchy consequences. The material on Fourier transforms is not exactly in the book (although we will revisit these integrals with residues—see Chapter 5 of the book).

Day 28: Wednesday, March 20. Today's material (which finished the Fourier transform estimate and started calculating a line integral that is rather harder than it looks) is not in the book, so you will need to rely on the notes.

Day 29: Friday, March 22. We proved the magnificent Cauchy integral formula, and there was much rejoicing in the land. The book states and proves this as Theorem 3.8.1. The book's version is vastly more general than ours in that it allows more arbitrary paths than circles, but at the cost of relying on the somewhat ambiguous notions of "positive orientation" and "interior." (Well, intuitively these notions are not ambiguous, but try casting them in exact mathematical language.) We will only use our version of the integral formula over circles—to get the "good stuff" that follows, circles are all we need. (Kindergarten geometry FTW!) Example 3.8.2 is very similar to our final (and only) example in class today. For Example 3.8.3, try rewriting the integrand using partial fraction and then use the Cauchy integral formula—no need for Cauchy's theorem "for multiply connected domains."

Day 30: Monday, March 25. The book states the generalized Cauchy integral formula in Theorem 3.8.6 and proves it using a more general differentiation under the integral argument (Lemma 3.8.4 and Theorem 3.8.5); that argument is more powerful than the one in the notes, but it requires more technical hypotheses on the integrand. Read Example 3.8.7 afterward. Liouville's theorem is Theorem 3.9.2. It relies on a more general version of the estimate that we proved; this is Theorem 3.9.1 (take $n = 2$ in that theorem for our estimate). Theorem 3.9.4 proves the fundamental theorem of algebra but relies on the notion of the limit of a function as $z \to \infty$ in $\mathbb{C}$ (Definition 2.2.10), which we did not develop.

Day 31: Wednesday, March 27. Our series expansion is Theorem 4.3.1, which relies on uniform convergence arguments and the Weierstrass M-test from Section 4.1 (in the book); we will not use those methods. Subsequent $+$-problems in the lecture notes will outline more self-contained methods to justify the interchange; the important thing is that you see the powerful role of the geometric series in conjunction with the picky hypotheses and precise structure of the Cauchy integral formula.

Day 32: Friday, March 29. You took Exam 2.

Day 33: Monday, April 1. Examples 4.3.4, 4.3.6, 4.3.7, 4.3.8, and 4.3.9 offer lots of practice with manipulating Taylor series and with obtaining Taylor series for new functions from known ones; we will continue practicing this next time. Definition 4.2.1 defines power series, and Example 4.2.2 tests the convergence of power series using the ratio and root tests. Theorem 4.2.5 states precisely the result on the radius of convergence, but doing this precisely requires the notion of a lim sup, which is too much real analysis for our class. You do not need to know this theorem as stated in the book, but you should read Definition 4.2.6 and look at Figure 4.5. Remark 4.3.3 discusses how to get a lower bound on the radius of convergence of a Taylor series.

Day 34: Wednesday, April 3. We did two excruciatingly detailed examples about Taylor series that further developed the recommended reading from Day 33. Problem 41 on pp. 260–261 contains further details about our Log example.

Day 35: Friday, April 5. Corollary 4.2.9 gives the formula for differentiating a power series, and Remark 4.3.3 revisits this formula for Taylor series. Examples 4.2.12 and 4.2.13 give more practice in recognizing a power series as the derivative of a function. The paragraphs at the bottom of p. 252 and at the start of Section 4.5 on p. 272 reinforce our remarks about factoring polynomials.

Day 36: Monday, April 8. We proved Theorem 4.5.2. Read Example 4.5.3 and maybe Example 4.3.10.

Day 37: Wednesday, April 10. We proved a more involved version of Theorem 4.5.4 (also with a different proof) and deduced from that the identity principle (Theorem 4.5.5).

Day 38: Friday, April 12. Example 4.5.6 shows how the identity principle allows one to extend functional equations from $\mathbb{R}$ into $\mathbb{C}$; we did this for a property of the logarithm. The paragraph just above that example is very important. Then we started discussing isolated singularities. Page 276 describes all three kinds and has three very illustrative figures. Example 4.5.9 (a), Proposition 4.5.10, Example 4.5.11, Theorem 4.5.12, and Example 4.5.13 all contain valuable information about removable singularities.

Day 39: Monday, April 15. We developed poles and essential singularities. All of the material on pp. 279–283 is worth reading. You don't have to read about singularities at ∞ on pp. 283–284. Ignore references to Laurent series for now; we will develop that shortly. Look at the picture of an annulus on p. 261.

Day 40: Wednesday, April 17. We stated, but did not prove, the very technical (and very important) theorem on the Laurent decomposition and the Laurent series. This appears as Theorem 4.4.1 in the textbook, which does not emphasize the functions f_r and f_R as we did. The proof is not easy; the textbook's relies on uniform convergence at several points, and the one in the appendices on…other stuff. However, the crux of the *existence* of the decomposition is establishing the identity (4.4.3) in the book; once you have that, the series results basically write themselves from geometric series arguments. More important for us is learning how to construct and manipulate Laurent series. Examples 4.4.2 through 4.4.7 are immensely worth reading. The identity (4.4.8) is helpful.

Day 41: Friday, April 19. We continued with Laurent examples and then talked about integrating a function analytic in an annulus over a closed path in that annulus. The result involves a very special Laurent coefficient and one other integral factor. Example 4.49 does this somewhat differently and just interchanges the whole Laurent series and the integral. There, that "other integral factor" does not appear (it's 1) because of the Death Star lemma.

Day 42: Monday, April 22. We discussed the winding number, which does not appear in the book, and then proved the almighty residue theorem, which appears as Theorem 5.1.2 in the book.

Day 43: Wednesday, April 24. Definition 5.1.1 defines residues. Proposition 5.1.3 gives some easy ways to calculate residues that do not involve the definition; see also Proposition 5.1.3, Theorem 5.1.6, and Lemma 5.7.1. In general, if you really need a residue, there's going to be a way to calculate it without the definition that involves an integral (Internet will help). Examples 5.1.4, 5.1.5, 5.1.7, and 5.1.8 give a bunch of "toy" line integrals that are evaluated using the residue theorem. (The point of these examples is not that the actual integral under consideration is hugely important; they're just ways to see the residue theorem in action.) Sections 5.2, 5.3, 5.4, and 5.5 offer a comprehensive, exhaustive, masterful treatment of various definite and improper integrals using residue methods. If you really need to calculate an integral using complex analysis, you can probably find ideas in there. Problem 24 in Section 5.4 offers a different approach to the Fourier integral that we are studying.

Day 44: Friday, April 26. We proved the open mapping theorem (Corollary 5.7.14) by means of the counting theorem (Theorems 5.7.2, 5.7.3, and 5.7.6) and Rouché's theorem (Theorem 5.7.9). All of these theorems have somewhat different proofs in the book. Section 5.7 is hugely worth reading for all of the consequences of the residue theorem that it presents; there is a wealth of information packed in here.