Reflection Activity

Submit responses to the following questions to the `Exam 2` slot on D2L by 11:59 pm on Monday, October 23. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Tell me everything you know about the RREF of a matrix. (Wait—I worry what you just read was "Tell me *a lot* about the RREF of a matrix." What I wrote was "Tell me *everything* you know about the RREF of a matrix.")

2. (Required.) Briefly explain why linear independence is useful.

3. (Required.) What have you found most difficult or confusing from Days 15 through 27 (which will be the content of the exam)? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

4. (Optional.) What would you like to discuss during our review in class on Wednesday, October 23? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

You will take Exam 2 on Friday, October 25. The exam will cover material discussed in class on Days 15 through 27. I will not ask questions about basis or dimension, since you haven't had a problem set on that yet. However, underlying ideas on spans and linear (in)dependence will certainly appear. All references below are to the daily log.

1. Give the precise definition of any concept appearing in the "Vocabulary from today" boxes in the daily log for Days 15 through 27 and provide examples and/or nonexamples as appropriate. Be sure to review those boxes again (i.e., right now) in case I updated them after posting the notes from that day. Vocabulary questions will be similar in style (if not content) to the ones from Vocabulary Quizzes 1 and 2.

I will not explicitly ask questions about vocabulary from Days 1 to 14 (unlike my policy on vocabulary quizzes)—for example, I won't ask you to define a linear operator. (If that's still a challenge, you have bigger things to worry about than this exam.) However, I will expect that you have fully mastered that vocabulary and are ready to use it—for example, I might ask you about invertibility of a linear operator, and that might require consideration of the earlier topics of injectivity and surjectivity.

2. Explain how operator composition motivates the definition of the matrix product AB. This is essentially the argument from Day 16.

3. Calculate the product of given matrices, or explain why it is undefined. Calculate the composition of given operators.

4. Determine if an operator or matrix is invertible and calculate its inverse.

5. Perform elementary row operations to transform a given (small) matrix into its RREF. Give the elementary matrix that performs each row operation. There may be multiple adequate orders in which you can do the row operations.

6. Decide if a given matrix is in RREF or not. If not, be able to explain all of the ways in which it fails to be in RREF. Construct examples of matrices that are/are not in RREF and explain the failures. Find all (small) matrices whose pivot columns are specified columns (as we did in Example 22.10). While I will not ask you to define RREF (in the sense of the four properties of Theorem 22.3), you should be able to recognize and deploy them instantly on any given matrix. You should be comfortable manipulating the RREF in its "factored" form from Theorem 22.6.

7. Decide if a given list of vectors is linearly independent or linearly dependent. You should be able both to prove and use the first five parts of Example 25.6.

8. You should be able to prove the following results: Theorem 15.6, Theorem 18.4, Theorem 24.2, Theorem 24.4, Lemma 25.1, Lemma 26.1, Theorem 26.5, Theorem 27.8, Corollary 27.9. You will not need to prove existence or uniqueness of the RREF.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?

5. Are you comfortable following the preparation instructions for the vocabulary quizzes for all of the vocabulary covered on this exam?