Reflection Activity

Submit responses to the following questions to the `Exam 2` slot on D2L by 11:59 pm on Monday, October 21. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Compare and contrast the behavior of solutions to the heat equation to those of the transport and wave equations. You might in particular consider properties such as existence and uniqueness of solutions, (in)finite propagation speed of initial data, and continuous dependence on initial conditions. Your response should make sense to someone who has taken Calculus III and knows what a partial derivative is but who hasn't taken this class.

2. (Required.) What have you found most difficult or confusing from Days 14 through 27 (which will be the content of the exam)? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Optional.) What would you like to discuss during our review in class on Wednesday, October 23? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook. At the same time, keep your requests concise—a list of 20 things suggests that you are not thinking too hard about the class.

Exam Content

You will take Exam 2 on Friday, October 25. The exam will cover material discussed in class on Days 14 through 27. Specifically, the exam will test your ability to do the following; numbered references below are to the daily log, unless otherwise specified.

1. Use D'Alembert's formula in connection with wave equation problems. You will not need to redevelop the formula, but you must have it memorized and ready to deploy. Failure to remember the formula was a problem for some of you on Exam 1.

2. Prove "compatibility conditions" for initial and boundary data for the semi-infinite and finite wave equations. See Problems 14.3 and 16.1 in the daily log.

3. Explain how to solve semi-infinite and finite wave equations by reflections or periodic extensions of odd reflections. You should be able to explain what the reflection is, what the periodic extension is, how you apply D'Alembert's formula to that data, and what is necessary for those extensions and reflections to be sufficiently differentiable.

4. Prove part (i) of Theorem 19.4 on continuous dependence on initial conditions for the infinite wave equation. I will not ask you to prove part (ii), although you should be able to explain that the finite wave equation has a "better" continuous dependence on initial conditions than the infinite equation.

5. Prove uniqueness for the finite wave equation. We did this on Day 19 and Theorem 1 on pp. 289–290 in the textbook has this all spelled out. You should be very comfortable with all of the mechanics of this proof.

6. Prove that a given function $f: \mathbb{R} \to \mathbb{C}$ is integrable and (maybe) calculate its integral. I will tell you if you need to use the definition (Definition 21.7) or if you can use the comparison test (Theorem 21.9). You should be comfortable with both approaches.

7. State the definition of the Fourier transform and prove that it converges for $f \in L^1$. Prove properties of the Fourier transform similar to those in Problems 23.2 and 23.3. I will not ask you about Fourier transform properties of the Gaussian $\mathcal{G}(x) = e^{-x^2}$.

8. Use properties of the Fourier transform to obtain the formal solution (22.2) for the heat equation.

9. Prove properties of the solution to the heat equation given in Theorem 24.5, like boundedness (Problem 24.4) or temporal limits at ∞ (Problem 24.6) or continuous dependence on initial conditions (Day 27). I will remind you of this solution formula, so you don't have to memorize that.

10. Prove uniqueness for the finite rod heat equation (Theorem 26.5, Theorem 1 on pp. 140–141 in the textbook). It is very likely that I will ask you to do this or the finite wave uniqueness. To test your mastery of these arguments, be able to compare and contrast them—both involve energy integrals, but the nuts and bolts of the calculus differ.

11. State (but not prove) the maximum principle and use it to find the maximum value of a solution to the heat equation on a given rectangle (see Example 2 on pp. 145–146 of the textbook and Problems 3 and 4 on p. 151 of the textbook).

12. I will not ask you about boundedness in finite time for the heat equation, i.e., you will not need to know how to prove Theorem 24.2.

13. Prove Theorem 27.3 on continuous dependence on initial conditions for the finite rod heat equation. Discuss what measurements of function size the norms $\|\cdot\|_\infty$, $\|\cdot\|_{L^1}$, and $\|\cdot\|_{L^2}$ give for continuous functions on a closed bounded interval.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed every problem set and checked your solutions carefully?

2. Have you completed every recommended problem from the problem sets?

3. Have you completed every problem in the daily log, other than those marked optional?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?