Reflection Activity

Submit responses to the following questions to the `Final Exam` slot on D2L by 11:59 pm on Saturday, November 30. This is much later than the due date in the syllabus; feel free to turn it in earlier. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. You are stranded on a desert island with a classmate. (The desert island is a metaphor for this class: you're not getting off until you pass.) Unfortunately, a coconut has fallen and bonked your classmate on the noggin, and now they can't remember anything from this course. Miraculously, they *can* remember all of Linear I and so have a lot of intuition for our more abstract work here.

Explain to them all of the terminology and mathematical machinery that goes into stating and proving the "range equals kernel perp" identity, which is equation (41.1) in the daily log. (I count at least seven things underlying just the statement alone that $\mathcal{T}(\mathcal{V}) = \ker(\mathcal{T}^*)^\perp$ for suitably nice $\mathcal{T}$.) Explain as well why this is an important result in our study of linear operators. Last, identify the concepts that you found most confusing or unsettling in the process of developing your explanation. It's possible some of these concepts will not be new things since Exam 2. Spend at least half an hour rereading your notes, the daily log, and the textbook to review those troublesome concepts, and describe how you feel now.

2. (Optional.) What would you like to discuss during our review in class on Monday, December 2? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

You will take Exam 2 on Friday, December 6, 1:00 pm–3:00 pm in our usual classroom (D250). The exam is cumulative and will cover the material tested on Exams 1 and 2, as well as the (more or less) new material detailed below. You should review the exam information documents for Exams 1 and 2, as that content is not listed here. All references below are to the daily log. *I may add more content to this list on Friday, November 22.*

1. Give the precise definition of any concept appearing in the "Vocabulary from today" boxes in the daily log for **all** days from the daily log and provide examples and/or nonexamples as appropriate. Be sure to review those boxes again (i.e., right now) in case I updated them after posting the notes from that day. Vocabulary questions will be similar in style (if not content) to the ones from the four vocabulary quizzes. I will repeat verbatim at least one question from at least one of the four quizzes, and I will ask about at least one concept that did not appear on any of the quizzes.

2. Explain how operator composition motivates the definition of the matrix product AB. This is essentially the argument from Day 16. *This was not really answered to my full satisfaction on Exam 2 by anyone, even if you got (generous) full credit. Review this carefully and be able to articulate it precisely.*

3. Find bases for the column and null spaces (ranges and kernels) of operators given by matrix-vector multiplication. Count dimensions of these spaces, too.

4. Explain why a given vector space is infinite-dimensional and decide if a given set is a basis or not. (It will help to be very comfortable with $\mathbb{R}^\infty$ and $\mathcal{C}([0,1])$.)

5. Determine if a given operation on a vector space is an inner product. I will not ask you to list out in the abstract the axioms for an inner product, but you should be able to check them from memory for a concrete operation on a specific vector space.

6. Compute the numerical values of inner products and norms of given vectors under specified inner products.

7. Explain why an orthonormal basis for a finite-dimensional subspace of an inner product space is preferable to any old basis. [Hint: *see the end of Day 36 in the daily log.*]

8. Determine the orthogonal complement of a given subspace of an inner product space.

9. Compute the adjoint of a given linear operator or explain why none exists.

10. Explain why the adjoint of matrix-vector multiplication is matrix-vector multiplication by the conjugate transpose (Example 40.1 in the daily log).

11. Prove the following results from the daily log: Lemma 28.4 (for very small, specified m and n), Lemma 29.1, Theorem 29.4, Theorem 29.7, Theorem 30.6, Theorem 31.1, Theorem 36.1, Theorem 36.7, Theorem 36.12, Theorem 36.13, Theorem 38.7, Theorem 38.13, Lemma 39.4, Lemma 41.2. You will not need to know how to prove Gram–Schmidt or do any Gram–Schmidt-type calculations (i.e., I will not ask you to actually find an orthonormal basis for a subspace).

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed every problem set and checked your solutions carefully?

2. Have you completed every recommended problem from the problem sets and all of the problems from the daily log, other than those marked optional?

3. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?

4. Are you comfortable following the preparation instructions for the vocabulary quizzes for all of the vocabulary covered on this exam?