Reflection Activity

Submit responses to the following questions to the `1` slot on D2L by 11:59 pm on February 10. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Let $A \in \mathbb{R}^{m \times m}$ and $\mathbf{b} \in \mathbb{R}^m$. How would you solve the linear system $A\mathbf{x} = \mathbf{b}$? Where and how might your methods fail? Discuss the role of the (in)dependence of the columns of A in your approach.

2. (Required.) What have you found most difficult or confusing in the course so far? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Optional.) What would you like to discuss during our review in class on Wednesday, February 12? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

4. (Optional.) What, if anything, do you want to change about how you are working in and approaching this course? How can I help?

Exam Content

The exam will cover material discussed in class on Days 1 through 14. Specifically, the exam will test your ability to do the following. All numbered references below are to the daily log.

1. Give precise definitions, examples, and/or nonexamples of the terms listed in the red vocabulary boxes at the start of each day's material in the daily log. Not all days have vocabulary, and not all important vocabulary terms are candidates for exam vocabulary questions—just those in the boxes. See the problem set instructions for how to prepare for vocabulary questions.

2. Calculate the product of a matrix and a vector or of two matrices, when such products are defined. I will specify if you need to use the "original definitions" or if dot products are allowed.

3. Determine if the columns of a matrix are dependent or independent.

4. Find the CR-factorization of a matrix. Such a matrix will be "easy" to scan and very similar to the examples from class and the textbook and the problems from the textbook.

5. Perform elimination and/or row interchanges (i.e., Gaussian elimination) on a matrix to reduce it to upper-triangular form. Provide the elimination and/or permutation matrices that perform these calculations; you will not need to compute the actual product of those matrices.

6. Solve an upper-triangular linear system by back-substitution or explain why no solution

exists.

7. Determine if a matrix is invertible, prove properties of invertible matrices, and calculate matrix inverses. We never developed an "easy" procedure for that last calculation, so I might ask you to calculate the inverse of an elimination or permutation matrix, which you can do by thinking about what such a matrix *does* (to a vector) rather than what it *is* (as an array of numbers).

8. Perform Gauss–Jordan elimination on an upper-triangular matrix (i.e., "eliminate upwards") to reduce it to the identity matrix, or a diagonal matrix if not the identity (Example 13.1).

9. I will not ask you to prove Theorem 14.6. However, you should know that each of those five statements implies the other, and what every word in each statement means. Working through Problem 14.7 will significantly reinforce your understanding of Theorem 14.6.

10. It is extremely likely that I will ask you to prove one of Theorems 4.7, 5.2, 8.4, 10.1, 12.6, 12.14, or 12.15.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above? It is extremely likely that I will put at least one of these problems (possibly one not assigned for a problem set) verbatim on the exam.

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?