Reflection Activity

Submit responses to the following questions to the 2 slot on D2L by 11:59 pm on Monday, March 24. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Tell me everything that you know about a matrix $A \in \mathbb{R}^{m\times n}$ if you know exactly the entries and structure of its RREF.

2. (Required.) What have you found most difficult or confusing in the course between the first exam and now? (Specifically, the material outlined below from Days 14–27.) Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Optional.) What would you like to discuss during our review in class on Wednesday, March 26? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

Days 15–27 in the daily log. Material on orthogonal complements from Day 27 will not be on the exam, so you can stop your review with Problem 27.11.

1. Give precise definitions, examples, and/or nonexamples of the terms listed in the red vocabulary boxes at the start of each day's material in the daily log for Days 15–27. I will not ask you explicitly for definitions, examples, and/or nonexamples regarding vocabulary from Days 1–14, but full knowledge of those terms will nonetheless be critical for your success. Not all days have vocabulary, and not all important vocabulary terms are candidates for exam vocabulary questions—just those in the boxes. See the problem set instructions for how to prepare for vocabulary questions.

2. Explain how to use an LU-factorization of a matrix $A \in \mathbb{R}^{m\times m}$, if one exists, to solve $A\mathbf{x} = \mathbf{b}$. Given an LU-factorization of a particular A, use it to solve $A\mathbf{x} = \mathbf{b}$ for a given $\mathbf{b}$. Briefly discuss the advantage of having an LU-factorization over other computational approaches for solving $A\mathbf{x} = \mathbf{b}$. You will not have to actually compute an LU-factorization from scratch; I'd give it to you.

3. Compute the null space of a given matrix and, ideally, express that null space as a column space. Determine a basis for that null space and its dimension.

4. Prove that a given set of vectors in $\mathbb{R}^p$ is a subspace of $\mathbb{R}^p$ or identify why it fails to be a subspace. Prove that the column space and null space of a matrix are indeed subspaces of $\mathbb{R}^p$ for appropriate p.

5. Perform elementary row operations to convert a matrix into its RREF. You should be able to write down the elementary matrices that perform each row operation, but you will not need to multiply them all together.

6. Determine if a given matrix is in RREF and otherwise explain why not. You should be familiar with all of the forms of the RREF from Theorem 19.9 and be able to write down concrete "small" versions for each case.

7. Explain the utility of having a basis for a subspace.

8. Given the RREF of a matrix, use that RREF to study the null space and column space. In particular, determine bases for and the dimensions of those spaces from the RREF. This will involve, in part, determining the pivot columns of both the RREF and the original matrix.

9. Compare and contrast the null spaces: define them, of what spaces $\mathbb{R}^p$ are they subspaces, and how do they encode information about solving $A\mathbf{x} = \mathbf{b}$? How do their dimensions interact?

10. Prove properties of the transpose of a matrix as outlined in Problems 26.5, 26.8, and 26.9 in the daily log.

11. Prove the following results (all numbering is from the daily log): Theorem 15.2 (only the proof given in the daily log, which is not the proof of the whole theorem), Theorem 21.8, Theorem 22.4, Theorem 22.7, Theorem 24.1, Theorem 24.5 (for p small—say I told you $p = 4$), Theorem 25.1 (this is really part (iii) of Example 24.12), Theorem 26.4 (you could use either the proof given right after the theorem or the proof that hinges on the calculation in equation (26.4)). I will not ask you to prove Theorem 20.3. I will not ask you to prove Theorem 27.9, but you should be able to explain what it says.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?