Reflection Activity

Submit responses to the following questions to the 2 slot on D2L by 11:59 pm on March 24. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Compare and contrast the behavior of solutions to the transport, wave, and heat equations. In your reflection for Exam 1, you contrasted the transport and wave equations; while your focus here should be on how heat is different, I encourage you to revisit your reflection for Exam 1 and my comments on that, and fill in any gaps in your understanding of transport and wave along the way. Your response should make sense to someone who has taken Calculus III and knows what a partial derivative is but who hasn't taken this class.

2. (Required.) What have you found most difficult or confusing in the material from Days 15–28 (as outlined below) that will be on the exam? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Required.) Submit drafts of solutions to two porfolio problems assigned on or before March 19. Please just include the drafts on a separate page within your larger response to this reflection activity.

4. (Optional.) What would you like to discuss during our review in class on Wednesday, March 26? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

The exam will test your understanding of material from Days 15 to 28 in the daily log. I expect that you have retained, and ideally improved, your understanding of the transport and wave equations from the Exam 1 coverage, and knowledge of those equations (especially as contrasted with the heat equation) may be necessary on this exam, too. All references below are to the daily log.

1. Explain how to use reflection and extension methods to extract from D'Alembert's formula a solution to the finite string wave equation (15.1). You do not need to know how to do Problem 15.2 (unless you want to do it for the portfolio), but you should be able to explain the overall strategy contained in that problem.

2. Use the definition of the improper integral to prove that an improper integral converges and calculate its exact value. Use the comparison test to prove that an improper integral converges and estimate its value. Explain what it means for a function to be in L^1; you may assume that the function is continuous, and I won't ask you to define precisely what a piecewise continuous function is (though you may need to integrate one nonetheless).

3. Find a formal solution to an ODE or PDE using the Fourier transform and Fourier inversion. In particular, derive the solution (20.1) to the heat equation. I will not ask you to continue that derivation to (20.3). You should be completely comfortable with the work from Days 27 and 28 on $y' = ay + f(x)$, in particular the succinct discussion on p. 135 that summarizes all of the hemming and hawing that preceded it.

4. Explain, but not prove, the result of Theorem 21.9. In particular, you should understand what it means for u to be bounded in finite time. I will remind you of the formula for H, but I expect that you know the formula for u given in that theorem.

5. Use the formula for u from Theorem 21.9 to prove properties of u, e.g., that u is bounded in finite time, or plain old bounded (depending on h), or that the heat equation exhibits infinite propagation speed (the paragraph after Problem 22.2).

6. State (but not prove) the maximum principle and use it to find extreme values of a solution to the heat equation.

7. Explain what the norms $\|\cdot\|_\infty$, $\|\cdot\|_{L^1}$, and $\|\cdot\|_{L^2}$ measure about a function. Compare and contrast them.

8. Compute a Fourier transform from the definition or from other transforms and properties of the transform (you should be extremely comfortable with the identity $\widehat{f'}(k) = ik\widehat{f}(k)$ and the results of Problems 20.5 and 20.6 and convolution).

9. Prove the following theorems from the daily log: Theorem 15.5, part (i) of Theorem 18.5, Theorem 24.2 (and supplement that with Problem 24.3), Theorem 25.1, part (ii) of Theorem 26.10.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?