Reflection Activity

This reflection has two parts. The first part is optional and is due to the `Final Exam Reflection: (optional) what do you want to review?` slot on D2L by 11:59 pm on Saturday, April 26. The second part is required if you want the 5 points of credit on the final exam and is due to the `Final Exam Reflection: required written response` slot on D2L by 11:59 pm on Friday, May 2.

1. (Optional, due on 4/26) What would you like to discuss during our review in class on Monday, April 28? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

2. (Required, due on 5/2) We have often remarked on how matrices both *encode* data and *act on* data. Describe how both actions have appeared throughout the course. You may want to consider topics such as the representation of linear systems as matrix-vector equations; matrix-vector and matrix-matrix multiplication; the RREF (and its computation); and matrix factorizations such as CR, LU, QR, and diagonalization. In your discussion, be sure to highlight both the *static* (encoding) and *dynamic* (acting on) aspects of matrices that you have observed. Your response should be accessible to a classmate who has made it this far in the course.

Exam Content

The final exam is cumulative and will cover material from throughout the course. All topics on the Exams 1 and 2 information documents may be tested in addition to the new material below. *This list has been updated to reflect our work on Friday, April 25.*

1. Give precise definitions, examples, and/or nonexamples of the terms listed in the red vocabulary boxes at the start of each day's material in the daily log for the entire course (Days 1–44). Not all days have vocabulary, and not all important vocabulary terms are candidates for exam vocabulary questions—just those in the boxes. See the problem set instructions for how to prepare for vocabulary questions.

2. Explain what the orthogonal decompositions $\mathbb{R}^n = \mathbf{N}(A) \oplus \mathbf{C}(A^\mathsf{T})$ and $\mathbb{R}^m = \mathbf{C}(A) \oplus \mathbf{N}(A^\mathsf{T})$ tell you. (Here $A \in \mathbb{R}^{m\times n}$.)

3. Develop the formula for the projection onto the column space of a matrix with full column rank. (This was outlined on Day 29.)

4. Explain when and how we would use least squares to study the problem $A\mathbf{x} = \mathbf{b}$.

5. Explain why an orthonormal basis is preferable to an "ordinary" basis and how orthonormality simplifies least squares.

6. Describe how the Gram–Schmidt procedure works and how you would perform it (Theorem 35.1 in the daily log). I will not actually ask you to do Gram–Schmidt on given vectors, as that is so prone to arithmetical error when done by hand that such a problem wouldn't necessarily reflect your true understanding of the process.

7. Explain how to use the Gram–Schmidt procedure to find the QR-factorization of a matrix with full column rank and how to use the QR-factorization in least squares. (This appeared on Day 36 in the daily log.) Again, I won't ask you to compute an actual QR-factorization by hand.

8. Find eigenvalues and eigenvectors of a matrix. While I may ask for eigeninformation of a matrix larger than 2×2, you will be able to get it without using determinants. You will only need to use determinants to find eigenvalues of a 2×2 matrix.

9. Calculate the determinant of a 3×3 matrix. (This will not involve finding eigenvalues, just getting $\det(A)$.)

10. Explain the utility of diagonalization in calculating matrix powers.

11. Explain why an orthogonally or unitarily diagonalizable matrix may be preferable to an "ordinarily" diagonalizable matrix.

12. Compute the determinant of a 2×2 or a 3×3 matrix. You may need the formula for this.

13. I will not ask about applications of eigentheory and diagonalization to ODE problems or Markov matrices.

14. I will not ask about the SVD, so you can stop your review with Problem 44.7 from the daily log.

15. If I ask a question about positive (semi)definite matrices, it will be similar to the problems from the daily log or the following from Section 6.3 of the textbook (which will also make you think about a bunch of related worthwhile concepts along the way): 1, 4, 5, 13, 14, 15, 18, 21, 23, 29, 46, 47.

16. Prove the following results from the daily log: Theorem 28.6, Theorem 30.10, Theorem 33.7, 34.5, Theorem 36.9, Theorem 37.11 (just the $j = 2$ case from the log), Theorem 38.1, Theorem 42.20, Theorem 42.21, Theorem 44.1.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?