Reflection Activity

Submit responses to the following questions to the slot on D2L by 11:59 pm on Saturday, April 26. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. We have often said throughout the course that integrals both *represent* functions and *measure or extract data about* functions. Give examples of how both behaviors of integrals have arisen in multiple contexts throughout the course. Your examples should go beyond the behaviors that you already saw in calculus via the FTC and average value, and you should discuss the behavior of integrals for at least two of the four major PDE (transport, wave, heat, Laplace) that we studied. Your discussion should be accessible to a classmate who has made it this far in the course.

2. (Optional.) What would you like to discuss during our review in class on Monday, April 28? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

The final exam is cumulative and will cover material from throughout the course. All topics on the Exams 1 and 2 information documents may be tested in addition to the new material below.

1. I will repeat at least one problem verbatim from Exam 1 and one problem verbatim from Exam 2.

2. I will include at least two (!)- or ($\star$) problems from the daily log; at least one of these will have previously been assigned on a problem set. No (+)-problems from the log will appear on the exam. I will not include any of the following problems: 2.4, 2.6, 4.2, 4.7, 5.7, 6.3, 7.8, 9.7, 10.8, 11.2, 11.8, 11.9, 11.10, 12.2, 13.1, 14.1, 14.2, 14.4, 14.5, 15.3, 16.1, 16.2, 16.3, 18.1, 18.2, 18.3, 18.4, 20.2, 20.4, 21.1, 21.5, 21.6, 21.7, 21.8, 22.7, 22.8, 22.9, 23.1, 24.1, 24.3, 24.4, 25.3, 25.7, 27.2, 27.9, 30.6, 31.3, 33.1, 33.2, 33.4, 34.1, 34.3, 34.4, 35.5, 36.10, 36.11, 37.5, 38.4, 38.5, 40.4, 41.1, 41.2, 41.3, 41.4, 41.5.

3. Sketch the argument that putting $u(x,t) = (H(\cdot,t) * f)(x)$ actually solves the heat equation. This was done on Day 34.

4. Find "radial" solutions to Laplace's equation in two and three dimensions.

5. State the weak and strong maximum principles for Laplace's equation, compare and contrast them, and use them to find extreme values of harmonic functions. You will not have to prove them.

6. The exam will not cover Fourier series.

7. Sketch the argument from Days 40 and 41 that a minimizer of the energy functional $\mathcal{E}$ is harmonic. I would remind you of the integration by parts identity in Lemma 40.3.

8. Prove the following results from the daily log: Theorem 33.5, Theorem 38.2 (I would remind you of Laplace's equation in polar coordinates and Lemma 38.3), Corollary 39.1.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the lecture notes corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?