Math relies on specialized, idiosyncratic vocabulary (and notation). It's hard to do anything or feel confident without understanding what the words mean, and often that meaning evolves and deepens as we learn more math. I suggest keeping a personalized glossary of the major concepts from our class. The next page of this document has an annotated guide for the concept of "continuous function" from calculus, which I chose just for the sake of having an example. The final page is a blank version of that guide.

The guide has six slots. The first two should be obvious: write down the concept that you want to study and the precise definition as given in the course. Then give one or more examples of that concept. Ideally these examples will be accessible and meaningful to you—when you think of the concept, these examples should immediately spring to mind. Don't feel that you have to use the examples from class or the textbook (although problems from the textbook may also furnish good examples). Try to keep all of the "auxiliary math" simple—make the numbers and formulas as easy for you as possible.

Some concepts may lend themselves to nonexamples: an "object" that has some underlying property of the concept but not all of the properties. In the annotated guide below, the concept is "continuous function," so the natural nonexample is "a function that is not continuous" (more concisely called a "discontinuous function," of course). Not every concept has a meaningfully related nonexample, however.

Last, pay attention to how the concept is used throughout the course starting from the time of its introduction (and maybe even before that). Why is this concept important? What is it good for? How has your understanding of, and maybe appreciation for, it changed? And is there anything else that you want to remember that didn't fit nicely into one of the categories above?

I suggest that you try to keep your records to one page per concept, so that your examples are as concise and efficient as possible, and so that your remarks on the concept's evolving role get to the point. However, don't feel that you strictly have to limit yourself to one page, or even a standard size piece of paper; maybe an index card (or one index card for each of the five areas after "Concept") will work best for you.

Concept. [Hopefully this is an accessible concept from calculus.] **Continuous function**

Definition. [This is the precise definition.] A function f defined on an interval I is **continuous** at a point a in I if $\lim_{x \to a} f(x) = f(a)$. If a is an endpoint of I, replace $\lim_{x \to a}$ with $\lim_{x \to a^+}$ or $\lim_{x \to a^-}$ as appropriate. If f is not continuous at a, then f is **discontinuous** at a.

Examples. [These examples feel "easy" and "obvious" to me because I can visualize them and the formulas are not complicated.]
• The function $f(x) = 1$ is continuous at each a in $(-\infty, \infty)$ because all constant functions are continuous on $(-\infty, \infty)$.
• The function $f(x) = 1/x$ is continuous at each a in $(0, \infty)$ because a rational function is continuous on any interval where its denominator is nonzero.

Nonexamples (if applicable). [I'm trying to break the definition in several ways. I'm not drawing pictures here to save space, but try drawing pictures whenever you can.]
• The function
$$f(x) = \begin{cases} 1, & -1 \le x < 0 \\ 2, & 0 \le x \le 1 \end{cases}$$
is defined on $[-1, 1]$ and discontinuous at 0 because $\lim_{x \to 0^-} f(x) = 1$ but $\lim_{x \to 0^+} f(x) = 2$, thus $\lim_{x \to 0} f(x)$ does not exist.
• The function
$$f(x) = \begin{cases} 1, & -1 \le x < 0 \\ 2, & x = 0 \\ 1, & 0 < x \le 1 \end{cases}$$
is defined on $[-1, 1]$ and discontinuous at 0 because $\lim_{x \to 0} f(x) = 1$ but $f(0) = 2$.
• The function $f(x) = \sqrt{x}$, defined on $[0, \infty)$, is discontinuous at -1 because f is not defined at -1.

Evolving role. [Roughly the order in which the ideas appear.]
• Encodes the idea that you can draw the graph without lifting your pencil.
• Intermediate value theorem: rigorous statement of what we intuitively expect to be true.
• Extreme value theorem: foundation for optimization.
• Every differentiable function is continuous: if the function is nice enough to have a slope, it should be continuous. (Not every continuous function is differentiable: corners.)
• Every continuous function is integrable: if the function is continuous, it is nice enough to have area under its graph. (Not every integrable function is continuous: piecewise continuity.)

Other remarks. [Thoughts that don't go anywhere else but that might be important.]
• Continuity of a function f at a point a requires checking (depending on how we count) three properties. First, a must belong to the domain of f, so $f(a)$ is defined. Second, the limit $\lim_{x \to a} f(x)$ must exist. (As noted above, we can replace $\lim_{x \to a}$ with $\lim_{x \to a^\pm}$ when a is an endpoint of the domain of f.) Third, the equality $\lim_{x \to a} f(x) = f(a)$ must be true.
• A function is not merely a formula; it is a way of relating elements from the domain to elements from the range. Specifying the domain of the function is essential to speaking coherently about continuity. If $f(x) = \sqrt{x}$ with domain $[0, \infty)$, then f is discontinuous at -1 because f is not defined there. However
$$g(x) = \begin{cases} 0, & x < 0 \\ \sqrt{x}, & 0 \le x \end{cases}$$
is continuous at -1.

Concept.

Definition.

Example.

Nonexample (if applicable).

Evolving role.

Other remarks.