Reflection Activity

Submit responses to the following questions to the 1 slot on D2L by 11:59 pm on Monday, February 16. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. Let $A \in \mathbb{R}^{m\times m}$ and $\mathbf{b} \in \mathbb{R}^m$. How do the concepts of dependence and independence help us understand the fundamental problem $A\mathbf{x} = \mathbf{b}$ and the important data stored in A? In your answer, you should discuss the definition of $A\mathbf{x}$, how that definition relates to the column space $\mathbf{C}(A)$ of A, what dependence and independence mean, how those two concepts relate to the null space $\mathbf{N}(A)$ of A, and what our various conjectures about dependence and independence are saying regarding the fundamental problem. You should also mention the pivot columns of A and the CR-factorization of A. Your answer should be accessible to a classmate who has been keeping up with the material and to you if you reread it an hour after you wrote it.

2. (Required.) What have you found most difficult or confusing in the course so far? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Optional.) What would you like to discuss during our review in class on Wednesday, February 18? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

4. (Optional, may give you some questions to ask me in the previous part.) I highly recommend filling in Problem 11.10 in the daily log as a major summary.

Exam Content

You will take 1 on Friday, February 20. The exam will test material covered in the daily log from Days 1 to 14. Specifically, the exam will test your ability to do the following.

1. Provide definitions, examples, and, as appropriate, nonexamples for all vocabulary indicated at the start of daily material in the log on Days 1 through 14. Not every day has required vocabulary, and not all definitions and terms within the daily log are candidates for the exam.

I have specified in the daily log which terms need nonexamples by marking them (N); not all do. You can probably find easy examples and (as needed) nonexamples within the daily log and the textbook, and I encourage you to memorize the ones that you find simplest and most meaningful and accessible. Your definitions should be so precise that I should be able to use them to decide whether any mathematical object I ever encounter does or does not meet the properties under consideration.

You may wish to fill out the vocabulary template provided on the course website and continue to update it as you experience the evolving role of prior concepts in the course.

2. Rewrite a system of linear equations as a matrix-vector equation $A\mathbf{x} = \mathbf{b}$.

3. Compute matrix-vector products, dot products of vectors, and matrix-matrix products. You may use dot products for matrix-vector and matrix-matrix products.

4. Explain what information column spaces and null spaces contain about the matrix-vector equation $A\mathbf{x} = \mathbf{b}$. Describe as precisely as possible the column and null spaces of specific matrices (such matrices would have some “simple” or “obvious” structure, as we still don’t have a universal algorithm for solving $A\mathbf{x} = \mathbf{b}$ yet).

5. Give examples of linear systems with a unique solution, no solutions, or infinitely many solutions.

6. Determine if the columns of a matrix are dependent or independent. Determine if a list of vectors is dependent or independent. There are many ways to do this, and I will not require that you use a particular method.

7. Explain every part of Conjecture 7.1 in the daily log. More generally, you should be able to explain the established connection among (in)dependence, null spaces, and uniqueness of solutions to $A\mathbf{x} = \mathbf{b}$ as well as the hypothesized connection among (in)dependence and existence of solutions to $A\mathbf{x} = \mathbf{b}$.

8. Find the pivot columns of a matrix (again, such a matrix on the exam would have some “simple” or “obvious” structure, as we still don’t have a universal algorithm for solving $A\mathbf{x} = \mathbf{b}$ yet) and determine the rank of a matrix. Explain how to determine the CR-factorization of a matrix, what information is contained in each factor C and R (in particular, what are the sizes of these matrices?), and compute the CR-factorization of a given matrix.

9. Use back-substitution to solve a linear system in the form $U\mathbf{x} = \mathbf{c}$, where U is upper-triangular.

10. Write down the elimination matrix E_{ij} that subtracts a multiple of one row of a vector (or matrix) from another. I will not ask you to use Gaussian elimination to reduce a matrix to upper-triangular form.

11. Prove the following results from the daily log: Theorem 8.9, Theorem 8.13, Theorem 8.21, Corollary 9.1, 11.6, parts (i) and (ii) of Theorem 12.11 (corresponding to steps 1 and 2 in the proof), Theorem 13.10.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the daily log corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully? In particular, while we did not have a quiz on the problems in Problem Set 4, those problems do treat material that will be on the exam.

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?