Reflection Activity

Submit responses to the following questions to the `Exam 2` slot on D2L by 11:59 pm on Monday, April 6. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Suppose that you know the RREF of a matrix $A \in \mathbb{R}^{m \times n}$ exactly. That is, you know every entry of $\mathsf{rref}(A)$, but you don't know any entry of A. Tell me everything that you know about A. (I worry that what you read was "Tell me a lot about A." What I wrote was "Tell me *everything* that you know about A.)

2. (Required.) What have you found most difficult or confusing in the course on Days 15 to 30? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Optional.) What would you like to discuss during our review in class on Wednesday, April 8? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

You will take Exam 2 on Friday, April 10. The exam will test material covered in the daily log from Days 15 to 30. Specifically, you should be able to do the following.

1. Provide definitions, examples, and, as appropriate, nonexamples for all vocabulary indicated at the start of daily material in the log on Days 15through 30. (While I will not explicitly ask about vocabulary from Days 1 through 14, you still need to be competent with that material.) Not every day has required vocabulary, and not all definitions and terms within the daily log are candidates for the exam. You may wish to fill out the vocabulary template provided on the course website and continue to update it as you experience the evolving role of prior concepts in the course.

I have specified in the daily log which terms need nonexamples by marking them (N); not all do. You can probably find easy examples and (as needed) nonexamples within the daily log and the textbook, and I encourage you to memorize the ones that you find simplest and most meaningful and accessible. Your definitions should be so precise that I should be able to use them to decide whether any mathematical object I ever encounter does or does not meet the properties under consideration.

2. Perform Gaussian elimination to reduce a square matrix to upper-triangular form. Perform Gauss–Jordan elimination to reduce a square upper-triangular matrix with nonzero diagonal entries to the identity matrix. Provide the elementary matrices that perform each step of elimination. You will not need to multiply out these elementary matrices.

3. Prove properties of invertible matrices—specifically, Theorems 15.12 and 16.6 and Problem 16.8 in the daily log (for that problem, use only the definition of invertible matrices, not

the invertible matrix theorem).

4. Use the equivalent conditions of the invertible matrix theorem to determine if a given matrix is invertible.

5. Use elimination to find all solutions to $A\mathbf{x} = \mathbf{b}$ for a given A. I may give you a particular $\mathbf{b}$ (e.g., $\mathbf{b} = (1, 2, 3, 4) \in \mathbb{R}^4$), or I may ask you to work with an arbitrary $\mathbf{b}$ (i.e., $\mathbf{b} = (b_1, b_2, b_3, b_4) \in \mathbb{R}^4$) and determine conditions on the components of $\mathbf{b}$ that guarantee the existence of a solution. Here you must use elimination, and I will not give credit for any other approach. (Sometimes students revert to trying to write one variable in terms of the others and not use any matrix methods, and that's always a disaster. Matrix methods are the whole point of the course!)

6. Determine the RREF of a matrix. (This is a byproduct of a correct solution to solving $A\mathbf{x} = \mathbf{b}$ via elimination.)

7. Given the RREF of a matrix, determine its CR-factorization of A and bases for its column and null spaces. I may give you just the RREF concretely and leave the columns of A labeled as $\mathbf{a}_1, \ldots, \mathbf{a}_n$, in which case some of your answers would involve $\mathbf{a}_1, \ldots, \mathbf{a}_n$ but nothing more specific than that.

8. Give the definition of a subspace and an example of a subset of $\mathbb{R}^p$ that is not a subspace. Prove that a given subset is a subspace (possibly by recognizing it as a column space). Give an example of a subspace of $\mathbb{R}^p$ with specified dimension.

9. Prove that the null space and the column space are subspaces. (I ask this on pretty much every Exam 2 and/or final exam in Linear I, and it always seems to be a struggle. The proofs are in the log, we did them in class, and you need to know them.)

10. Explain the utility of having a basis for a subspace and prove Theorem 26.12 in the daily log.

11. State the rank–nullity theorem and use it to calculate dimensions of null or column spaces.

12. Prove Theorem 28.11 in the daily log. Either the proof from Day 28 or from Day 29, the latter using (29.1) as a critical step, is acceptable.

13. Given $A \in \mathbb{R}^{m\times n}$, explain what the abbreviation $\mathbb{R}^n = \mathbf{N}(A) \oplus \mathbf{C}(A^\mathsf{T})$ means. This is Theorem 30.5 in the daily log.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the daily log corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?

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