Reflection Activity

Submit responses to the following questions to the `Final Exam` slot on D2L by 11:59 pm on Saturday, May 2. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Strang concludes Chapter 4 in the textbook with a short section titled "Thoughts on the Victory of Orthogonality." (This is p. 197, and you may want to read it as outlined at the end of Day 39 in the daily log.) Explain in your own words this "victory." You should discuss least squares, orthogonal projections, orthonormal bases, and the incredibly satisfying feeling of knowing how exactly column and null spaces "fit into" the whole of Euclidean space.

2. (Required.) What have you found most difficult or confusing in the course on Days 1 to 44? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Optional.) What would you like to discuss during our review in class on Monday, May 4? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

You will take Final Exam on Friday, May 8, 1:00 pm–3:00 pm. The exam will test material covered in the daily log from Days 1 to 44.

1. Provide definitions, examples, and, as appropriate, nonexamples for all vocabulary indicated at the start of daily material in the log on Days 1through 44. Not every day has required vocabulary, and not all definitions and terms within the daily log are candidates for the exam. You may wish to fill out the vocabulary template provided on the course website and continue to update it as you experience the evolving role of prior concepts in the course.

I have specified in the daily log which terms need nonexamples by marking them (N); not all do. You can probably find easy examples and (as needed) nonexamples within the daily log and the textbook, and I encourage you to memorize the ones that you find simplest and most meaningful and accessible. Your definitions should be so precise that I should be able to use them to decide whether any mathematical object I ever encounter does or does not meet the properties under consideration.

2. You should be able to do all of the content of the exam information documents for Exams 1 and 2. If I ask you to find a CR-factorization of a matrix, you can use the RREF, but it may be just as efficient to do it the "original" way using the linear independence lemma.

3. Discuss, compare, and contrast the three major matrix factorizations that we have studied: CR, QR, and diagonalization. To what kinds of matrices do they apply? Under what conditions do they exist? What information is contained in each of the factors? How are these factorizations used to understand matrices, including but not limited to the problem $A\mathbf{x} = \mathbf{b}$?

4. Use the characterization $\mathbf{C}(A) = \mathbf{N}(A^\mathsf{T})^\perp$ to determine if the problem $A\mathbf{x} = \mathbf{b}$ has a solution.

5. Prove Theorem 36.1 (least squares) and explain why the least squares problem $A\widehat{\mathbf{x}} = P_A\mathbf{b}$ is the best approximate problem to $A\mathbf{x} = \mathbf{b}$ when $\mathbf{b} \notin \mathbf{C}(A)$. I will not ask you to actually compute $P_A\mathbf{b}$ or solve $A\widehat{\mathbf{x}} = P_A\mathbf{b}$ for a specific A or $\mathbf{b}$. Explain why the normal equation $A^\mathsf{T}A\widehat{\mathbf{x}} = A^\mathsf{T}\mathbf{b}$ is equivalent to the least squares problem and why you might prefer solving it to the original least squares problem. Explain how the normal equation simplifies if A is orthogonal or if you have a QR-factorization of A; for the latter, you should explain how the least squares problem is equivalent to $R\widehat{\mathbf{x}} = Q^\mathsf{T}\mathbf{b}$ and why that is the most preferable problem to have.

6. Explain the utility of orthonormal bases and how to construct them (i.e., give pseudocode for Gram–Schmidt). Prove Theorem 37.3 and Theorem 37.15 from the daily log.

7. Compute eigenvalues and eigenvectors for 2×2 matrices using the characteristic equation or, more generally, for upper-triangular matrices. Problems involving eigenvalues and eigenvectors will be very similar to the problems from the daily log and Problem Set 12.

8. I expect that you have attempted all of the (!)- and ($\star$)-problems in the daily log as you have worked your way through the course. However, you do not need to have done the following (!)- and ($\star$)-problems in preparation for the final: 8.8, 8.18, 9.4, 10.1, 11.9, 13.3, 13.4, 13.5, 13.6, 13.7, 15.3, 18.6, 18.7, 19.5, 19.6, 19.8, 19.11, 20.4, 23.11, 24.7, 26.6, 27.3, 27.7, 27.8, 27.9, 27.10, 28.19, 30.8, 32.9, 33.4, 33.6, 33.7, 33.8, 33.14, 36.7, 36.9, 37.7, 37.18, 38.1, 38.3, 39.2. I do not expect that you have looked at any (+)-problems.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the daily log corresponding to the material above (except for the ones that I told you that you could skip)?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?

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