Reflection Activity

Submit responses to the following questions to the `Final Exam` slot on D2L by 11:59 pm on Saturday, May 2. Any cogent response will earn you 5 points on the exam; you can earn the other 95 points on the exam itself.

1. (Required.) Discuss the evolving role of eigenvalues in this course. What information, if any, about a linear operator do they encode? In what situations are eigenvalues more useful than others? How do eigenvalues interact with inner products and finite-dimensionality? Depending on how much your first course in linear algebra covered eigenvalues, describe how (if at all) our treatment has been different.

2. (Required.) What have you found most difficult or confusing in the course on Days 1 to 44? Write it down explicitly. Then think hard about this concept for at least half an hour—go back over your notes, the daily log, and the textbook and reread and rework material related to this sticky topic. How do you feel now?

3. (Optional.) What would you like to discuss during our review in class on Monday, May 4? Please be as specific as possible and, if you can, point to numbered items in the daily log, problems from problem sets, or content in the textbook.

Exam Content

You will take Final Exam on Monday, May 11, 1:00 pm–3:00 pm. The exam will test material covered in the daily log from Days 1 to 44.

1. Provide definitions, examples, and, as appropriate, nonexamples for all vocabulary indicated at the start of daily material in the log on Days 1through 44. Not every day has required vocabulary, and not all definitions and terms within the daily log are candidates for the exam. You may wish to fill out the vocabulary template provided on the course website and continue to update it as you experience the evolving role of prior concepts in the course.

I have specified in the daily log which terms need nonexamples by marking them (N); not all do. You can probably find easy examples and (as needed) nonexamples within the daily log and the textbook, and I encourage you to memorize the ones that you find simplest and most meaningful and accessible. Your definitions should be so precise that I should be able to use them to decide whether any mathematical object I ever encounter does or does not meet the properties under consideration.

2. You should be able to do all of the content of the exam information documents for Exams 1 and 2.

3. Describe the orthogonal complement of a given subspace of a specific inner product space. Prove Lemma 30.7, Theorem 30.10, Theorem 31.11, and Theorem 32.2 in the daily log. Find the best approximation to a given vector within a specific subspace of a particular inner product space using Theorem 32.2 and, possibly, Theorem 36.8.

4. Explain the role of an adjoint and the four fundamental subspaces associated with a linear operator in (possibly) providing a geometric characterization of the operator's range.

5. Find a formula for the adjoint of a specific linear operator on a given inner product space. Prove Theorem 33.6, Theorem 36.1, and Lemma 36.7 in the daily log.

6. Find the matrix representation of an operator between two finite-dimensional spaces with respect to given bases for those spaces.

7. Determine if a given linear operator is (unitarily) diagonalizable and, if so, find a diagonal factorization.

8. Prove properties of upper-triangular operators as detailed in the results and problems from Day 38 in the daily log. I will not ask you to prove Theorem 39.4, but you should know its statement.

9. Prove that unitarily diagonalizable operators are normal. You should be able to state Schur's theorem and the full spectral theorems (real and complex), but I will not ask you to prove any of them.

10. Explain how the singular value decomposition is a substitute for diagonalization when diagonalization is impossible, and how the SVD is possibly a better alternative for approximation purposes. I won't ask you to prove Theorem 42.4 from the daily log, but understanding its proof inside and out is a good way to test your mastery of most of the course's finite-dimensional material. Do you really understand every word and every auxiliary result?

11. I expect that you have attempted all of the (!)- and ($\star$)-problems in the daily log as you have worked your way through the course. However, you do not need to have done the following (!)- and ($\star$)-problems in preparation for the final: 1.7, 2.5, 2.8, 2.9, 2.13, 3.7, 11.1, 11.3, 11.4, 11.12, 13.15, 13.16, 15.9, 16.3, 18.2, 18.14, 21.2, 22.3, 24.8, 24.9, 29.4, 29.6, 29.7, 29.12, 31.2, 31.4, 31.8, 31.10, 31.15, 32.1, 32.8, 33.8, 34.5, 34.10, 36.6, 36.10, 37.3, 37.11, 39.5, 40.5, 40.6, 40.11, 40.14, 40.17, 41.6, 41.8 41.11, 42.2. I do not expect that you have looked at any (+)-problems.

A natural question is how many problems will be on the exam. A numerical answer to this question that does not also discuss the length and difficulty of each problem (which would, more or less, require disclosing the content of each problem) will tell you very little. I expect that most students will need the full allotted time to complete an exam. There is definitely nothing wrong with you if the exam takes you all of the available time.

How to Prepare

Here are some questions for your consideration.

1. Have you completed all of the (!)- and ($\star$)-problems in the daily log corresponding to the material above?

2. Have you completed every problem set and checked your solutions carefully?

3. Have you completed every recommended problem from the problem sets?

4. Can you do all these problems with minimal reference to your notes, my notes, the textbook, or any other source?

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